

MATH II

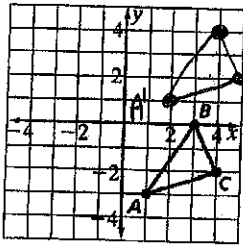
Name: Rey Date: _____ Period _____ Class _____

Transformational Geometry
 A transformation is a change in the size, location, or orientation of a figure.

A translation is a transformation which moves each point of a figure the same distance and in the same direction.

The resulting figure after a transformation is called the image of the original figure.

EXAMPLE 1:
 $\triangle ABC$ is translated 1 unit right and 4 units up. Draw the image $\triangle A'B'C'$.
 What are the coordinates of:



- $A (1, -3) \rightarrow A' (2, 1)$
- $B (3, 0) \rightarrow B' (4, 4)$
- $C (4, -2) \rightarrow C' (5, 2)$

From EXAMPLE 1, $\triangle ABC \rightarrow \triangle A'B'C'$
 As a general rule this translation could be written as
 $(x, y) \rightarrow (x + \underline{\quad}, y + \underline{\quad})$.

EXAMPLE 2

$\triangle JKL$ has coordinates $J(0, 2)$, $K(3, 4)$, and $L(5, 1)$.
 a) Draw $\triangle JKL$. b) Draw the image $\triangle J'K'L'$ after a translation of 4 units to the left and 5 units up. Label the triangle. What are the coordinates of:

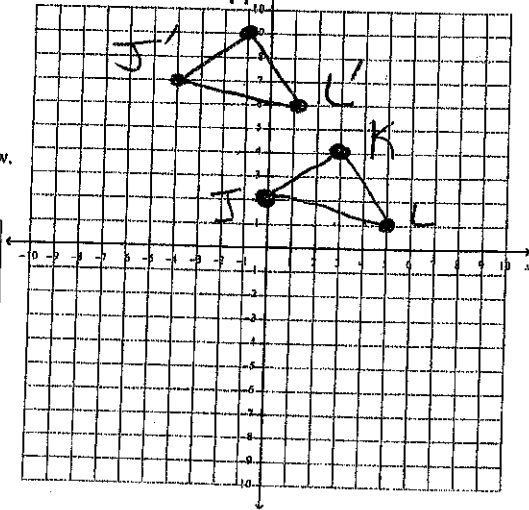
- $J (0, 2) \rightarrow J' (-4, 7)$
- $K (3, 4) \rightarrow K' (-1, 9)$
- $L (5, 1) \rightarrow L' (1, 6)$

Rule: $(x, y) \rightarrow (x - 4, y + 5)$

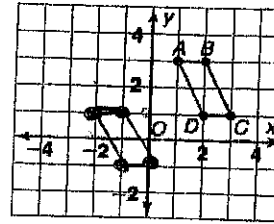
Tell me more about this figure, is it congruent or similar? Explain how you know.

Translation Location

	Add	Subtract
x coordinate	right	left
y coordinate	up	down



left 3 units, down 2 units



2 a) Use arrow notation to write a rule for the given translation. $(x, y) \rightarrow (x - 3, y - 2)$

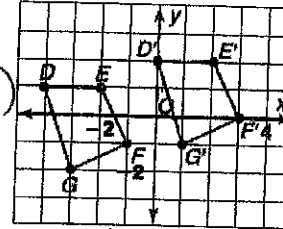
b) Graph and label the image after the translation.

c) Name the coordinates of the image.

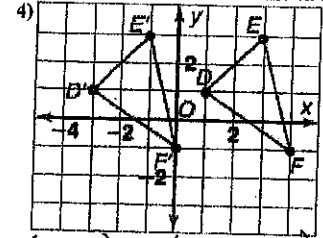
- $A' (-2, 1)$ $B' (-1, 1)$
- $C' (0, -1)$ $D' (-1, -1)$

In questions 3 and 4 below, use arrow notation to write a rule that describes the translation shown on the graph.

3)



$(x, y) \rightarrow (x + 4, y + 4)$



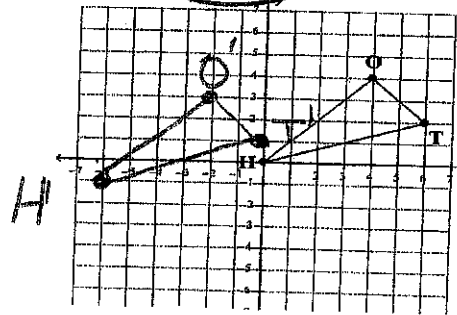
$(x, y) \rightarrow (x - 4, y)$

5) MULTIPLE CHOICE: Write a description of the rule $(x, y) \rightarrow (x - 7, y + 4)$.

- (a) translation 7 units to the right and 4 units up
- (b) translation 7 units to the left and 4 units down
- (c) translation 7 units to the right and 4 units down
- (d) translation 7 units to the left and 4 units up

Classwork/Homework/More Practice/Graded Work

1. Draw the translation of the triangle HOT six units left and one unit down. Label the image $H'O'T'$. Is the image similar or congruent? How do you know?

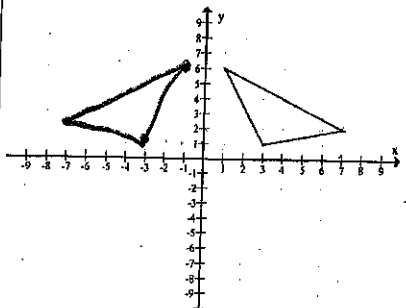


Same shape
Same size

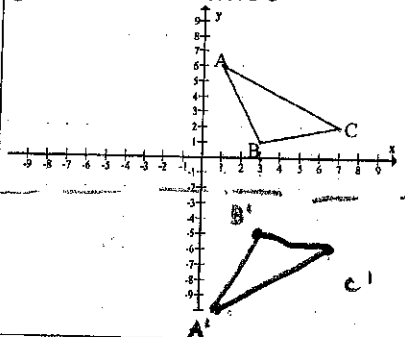
Name: _____

Reflections Worksheet

1. Give the coordinates of ABC. Reflect over the y-axis. Then give the coordinates of A'B'C'



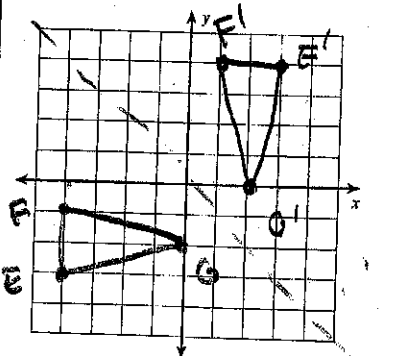
2. Give the coordinates of ABC. Reflect over the line horizontal line $y = -2$. Then give the coordinates of A'B'C'



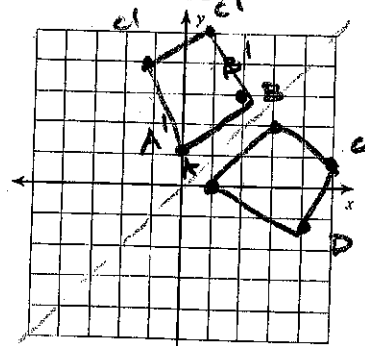
1.
A(1, 6) A'(-1, 6)
B(3, 1) B'(-3, 1)
C(7, 2) C'(-7, 2)

2.
A(1, 6) A'(1, -10)
B(3, 1) B'(3, -5)
C(7, 2) C'(7, -6)

3. Plot E(-4, -3), F(-4, -1), G(0, -2). Reflect across the line $y = -x$. Give the coordinates of E'F'G'.



4. Plot A(1, 0), B(3, 2), C(5, 1), D(4, -1). Reflect across the line $y = x$. Give the coordinates of A'B'C'D'.



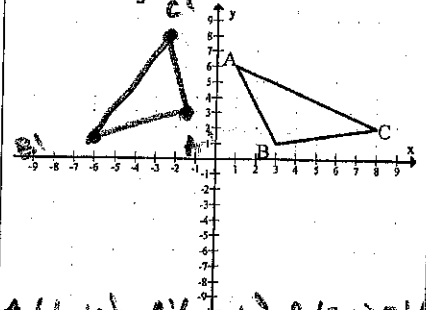
3. E'(3, 4) F'(1, 4) G'(2, 2)

4. A'(0, 1) B'(2, 3) C'(1, 5) D'(-1, 4)

Rotations Worksheet

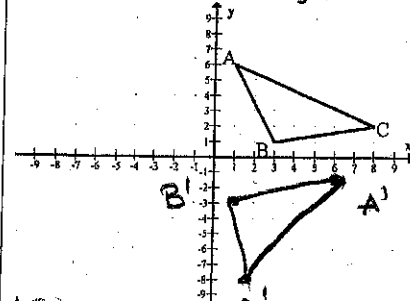
For #1-4, draw the triangle after each transformation and give the coordinates of A', B' and C'.

1. Rotate the triangle 90° counterclockwise about the origin.

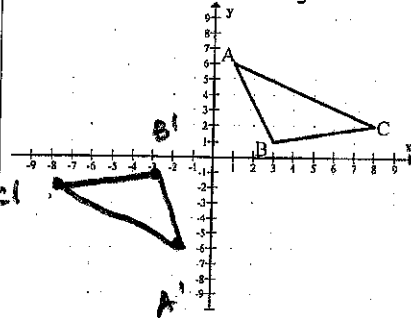


A(1, 6) A'(-6, 1) B(3, 1) B'(-1, 3)

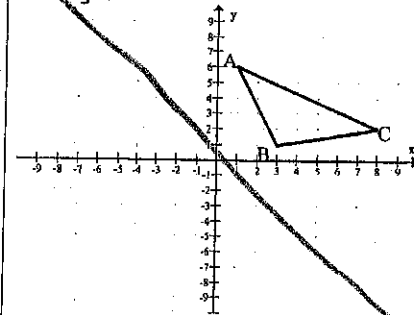
2. Rotate the triangle 270° counterclockwise about the origin. **90 CW**



3. Rotate the triangle 180° counterclockwise about the origin.



4. Rotate the triangle 90° clockwise about the origin.



Complete.

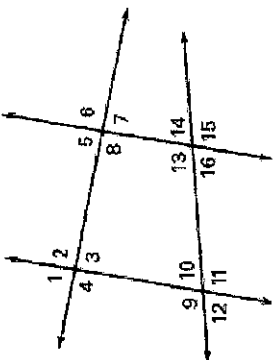
5. Give the coordinates of D(-2, -4) after a 270° counterclockwise rotation about the origin. **90 CW**

D'(-4, 2)

6. Give the coordinates of E(-4, 5) after a 180° counterclockwise rotation about the origin.

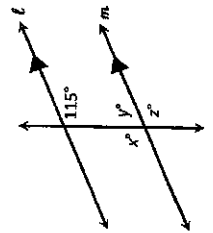
E'(4, -5)

Classify each angle pair as corresponding, alternate interior, alternate exterior, consecutive interior, or consecutive exterior.



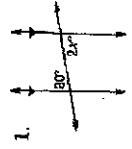
- a) $\angle 1$ and $\angle 9$ CORR
- b) $\angle 8$ and $\angle 13$ SSI
- c) $\angle 6$ and $\angle 16$ AEA
- d) $\angle 4$ and $\angle 10$ AIA
- e) $\angle 8$ and $\angle 16$ CORR
- f) $\angle 10$ and $\angle 13$ SSI

Discovery: Lines l and m are parallel. Note: Parallel lines are distinguished by a matching set of arrows on the lines that are parallel. Find the measure of the missing angles by using transparent paper. Then, let's go back and fill in the theorems.

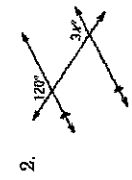


Key Question: If $x = 115^\circ$, is it possible for y to equal 115° ?
 No, SSI are supplementary

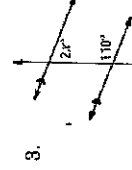
For the following diagrams, state the type of angles that are given, state their relationship, and then find x .



1. AIA \cong
 $2x = 80$
 $x = 40$

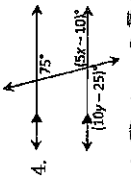


2. CORR \cong
 $3x = 120$
 $x = 40$

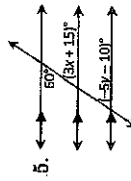


3. SSI
 180
 $2x + 110 = 180$
 $2x = 70$
 $x = 35$

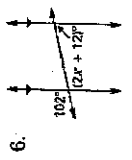
Find the missing variables.



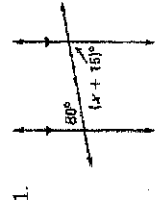
4. $75 + 5x - 10 = 180$
 $5x + 45 = 180$
 $5x = 135$
 $x = 27$
 $10y - 25 = 105$
 $10y = 130$
 $y = 13$



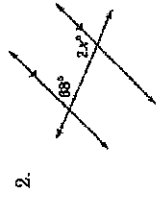
5. $60 = 3x + 15$
 $45 = 3x$
 $15 = x$
 $60 = 3y - 10$
 $70 = 3y$
 $23.33 = y$



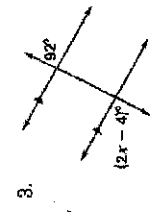
6. $102 + 2x + 12 = 180$
 $2x + 114 = 180$
 $2x = 66$
 $x = 33$



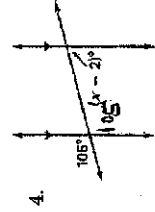
1. AIA \cong
 $80 = x + 15$
 $x = 65$



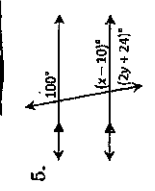
2. SSI 180
 $68 + 2x = 180$
 $2x = 112$
 $x = 56$



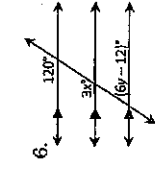
3. AEA \cong
 $2x - 4 = 92$
 $2x = 96$
 $x = 48$



4. SSI 180
 $105 + x - 2 = 180$
 $x + 103 = 180$
 $x = 77$



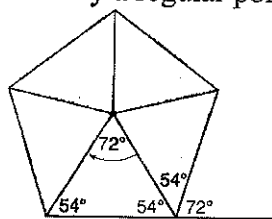
5. CORR \cong
 $100 = x - 10$
 $110 = x$
 $y = 28$



6. CORR \cong
 $3x = 120$
 $x = 40$
 $SS1 180$
 $120 - 12 + 6y = 180$
 $108 + 6y = 180$
 $6y = 72$
 $y = 12$

1 ANS: 2

Segments drawn from the center of the regular pentagon bisect each angle of the pentagon, and create five isosceles triangles as shown in the diagram below. Since each exterior angle equals the angles formed by the segments drawn from the center of the regular pentagon, the minimum degrees necessary to carry a regular polygon onto itself are equal to the measure of an exterior angle of the regular polygon.



$$\frac{360}{5} = 72.$$

REF: spr1402geo

2 ANS: 3

$$\frac{360^\circ}{5} = 72^\circ \quad 216^\circ \text{ is a multiple of } 72^\circ$$

REF: 061819geo

3 ANS: 1

$$\frac{360^\circ}{45^\circ} = 8$$

REF: 061510geo

4 ANS: 4

$$\frac{360^\circ}{10} = 36^\circ \quad 252^\circ \text{ is a multiple of } 36^\circ$$

REF: 011717geo

5 ANS: 4

$$\frac{360^\circ}{10} = 36^\circ \quad 252^\circ \text{ is a multiple of } 36^\circ$$

REF: 081722geo

6 ANS: 3

REF: 011815geo

7 ANS: 1

REF: 061707geo

8 ANS: 3

REF: 011904geo

9 ANS: 1

REF: 081505geo

10 ANS: 3

The x -axis and line $x = 4$ are lines of symmetry and $(4, 0)$ is a point of symmetry.

REF: 081706geo

11 ANS: 3

REF: 081817geo

12 ANS:

$$\frac{360}{6} = 60$$

REF: 081627geo

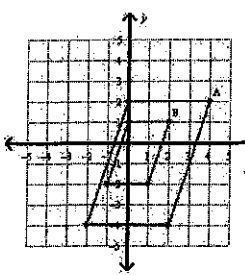
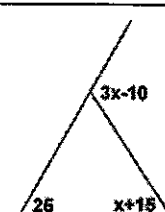
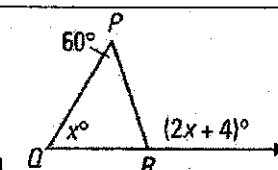
- [1] B
- [2] D
- [3] C
- [4] A
- [5] A
- [6] C
- [7] D
- [8] B
- [9] reflection
- [10] translation
- [11] rotation
- [12] rotation
- [13] translation
- [14] rotation
- [15] translation
- [16] glide reflection

Name: _____

Date: _____

Period _____

What you need to know & be able to do	Things to remember	Problem	Problem
Translations	<ul style="list-style-type: none"> Find the new coordinates by adding/subtracting the given value. Find the pre-image by doing the OPPOSITE. A translation is a rigid motion which means the pre-image and image are congruent 	<p>1. Translate the following points by the rule: $(x,y) \rightarrow (x+1, y-4)$</p> <p>$D(-5, 2) \rightarrow D'(-4, -2)$</p> <p>$O(-4, 5) \rightarrow O'(-3, 1)$</p> <p>$G(-1, 1) \rightarrow G'(0, -3)$</p> <p>$S(-4, -2) \rightarrow S'(-3, -6)$</p>	<p>2. Translation: $(x, y) \rightarrow (x-2, y-6)$</p> <p>Graph pre-image and image. $C(3, 2)$ $A(2, 4)$ $T(3, 5)$ $S(5, 2)$</p>
Reflections	<ul style="list-style-type: none"> Reflect over x-axis: $(x, -y)$ Reflect over y-axis: $(-x, y)$ Reflect across $y = x$ (switch x and y) Reflect across $y = -x$ (switch x and y AND change their signs) 	<p>3. Reflect across $y = x$ $(x, y) \rightarrow (y, x)$</p> <p>$E(-1, -5)$ $F(5, -1)$ $G(-1, -2)$ $G'(-2, -1)$ $H(0, 4)$ $H'(4, 0)$</p>	<p>4. Reflect across $y = -x$ then reflect across the y-axis</p> <p>$W(-3, -1)$ $W'(1, 3)$ $W''(-1, 3)$ $B(-5, 2)$ $B'(2, 5)$ $B''(-2, 5)$ $E(-3, -3)$ $E'(3, 3)$ $E''(-3, 3)$</p>
Rotations	<ul style="list-style-type: none"> 90CW/270CCW: $(y, -x)$ 180: $(-x, -y)$ 90CCW/270CW: $(-y, x)$ "drive the car": the fist that goes over the other is the sign that changes; switch the order. 	<p>5. Rotate the figure 90 CW $(x, y) \rightarrow (y, -x)$</p> <p>$T(0, 5)$ $T'(5, 0)$ $W(-2, 1)$ $W'(1, 2)$ $E(3, 1)$ $E'(1, -3)$</p>	<p>6. Rotate the figure 90 CCW $(x, y) \rightarrow (-y, x)$</p> <p>$D(-1, 3)$ $D'(-3, -1)$ $Y(-1, 1)$ $Y'(-1, -1)$ $L(3, 4)$ $L'(-4, 3)$</p>
Dilations	<ul style="list-style-type: none"> Multiply the coordinates by the given scale factor (k) Pre-image and image are NOT congruent; they are similar. Dilations are NOT rigid motions 	<p>X A. Find the coordinates of the new vertices of the image that has been dilated by a factor of 5.</p> <p>$R(-4, 5) \rightarrow R'(-20, 25)$</p> <p>$A(-1, 1) \rightarrow A'(-5, 5)$</p> <p>$T(-4, -2) \rightarrow T'(-20, -10)$</p>	<p>X Find the scale factor of the outside image if the inside figure is the pre-image. (smaller to larger) >>> see next page</p>

		<p>X Find the coordinates of the new vertices of the image that has been dilated by a factor of $\frac{1}{2}$.</p> <p>$U(2, 4) \rightarrow U'(1, 2)$ $R(4, -6) \rightarrow R'(2, -3)$ $P(-2, 2) \rightarrow P'(-1, 1)$</p>	 <p>$A(4, 2)$ $B(2, 1)$ Dilation of 2</p>
<p>Multiple Transformations,</p>	<ul style="list-style-type: none"> ORDER IS IMPORTANT Use the <i>previous</i> ordered pairs to do the next transformation. 	<p>9. Given the points $M(-3, 1)$ $S(5, -2)$</p> <p>Translate: $(x - 3, y + 2)$ Reflect: over y-axis</p> <p>$M' \rightarrow (-6, 3)$ $S' \rightarrow (2, 0)$ $M'' \rightarrow (6, 3)$ $S'' \rightarrow (-2, 0)$</p>	<p>10. Given the points $K(0, -4)$ $P(-6, -3)$ $R(1, 2)$</p> <p>Reflect: over the x-axis Rotate: 270 CCW 90 CW</p> <p>$K' \rightarrow (0, 4)$ $P' \rightarrow (-6, 3)$ $R' \rightarrow (1, -2)$ $K'' \rightarrow (4, 0)$ $P'' \rightarrow (3, 6)$ $R'' \rightarrow (2, 1)$</p>
<p>Angles of a triangle</p>	<ul style="list-style-type: none"> Angles add up to 180 The exterior angle of a triangle is equal to the sum of the 2 remote interior angles 	<p>11. The angles of a triangle measure $x+14$, $4x-2$, and $5x+8$. Solve for x and find the 3 angle measures.</p> <p>$x = 16$ Angles <u>30, 62, 92</u></p>	<p>12. Given the sides lengths, find the interval of the 3rd side.</p> <p>a. 5 and 8 $3 < x < 13$ b. 10 and 11 $1 < x < 21$</p>
		<p>13.</p>  <p>$x = 25$</p>	<p>14.</p>  <p>$x = 64$</p>

⑪ $\underline{x} + \underline{14} + \underline{4x} - \underline{2} + \underline{5x} + \underline{8} = 180$

$10x + 20 = 180$
 $10x = 160$
 $x = 16$

⑬ $25 + x + 15 = 3x - 10$

$40 + x = 3x - 10$
 $50 = 2x$
 $\boxed{25 = x}$

⑭ $x + 60 = 2x - 4$
 $64 = x$

16

$$2x + 20 + 3x + 45$$

$$5x + 65 = 180$$

$$5x = 115$$

$$x = 23$$

20 $82 = 2x + 10$

$$72 = 2x$$

$$36 = x$$

- Parallel lines cut by a transversal forms congruent and supplementary angles

Special angle relationships

- Angle relationships can be vertical, adjacent, alternate interior, alternate exterior, corresponding, same-side interior, and same-side exterior.

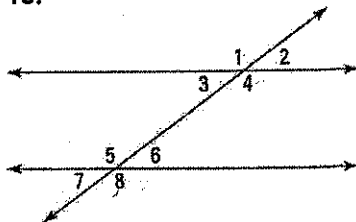
21 $5x + 10 + 3x + 2$

$$8x + 12 = 180$$

$$8x = 168$$

$$x = 21$$

15.



Congruent angles:

1 and 4, 5, 8

2 and 3, 6, 7

Supplementary angles:

Angle 1 and 3, 7, 2

Angle 2 and 1, 4, 8

16. If $\angle 3 = 2x + 20$ and $\angle 5 = 3x + 45$, solve for x and find angles 5 and 3.

$$x = 23$$

$$\angle 3 = 66$$

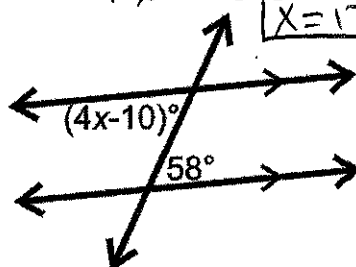
$$\angle 5 = 114$$

17. a. Solve for x .

$$4x - 10 = 58$$

$$4x = 68$$

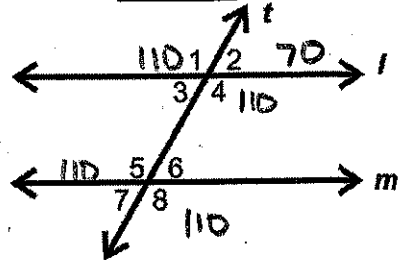
$$x = 17$$



18. If $m\angle 5 = 110$, find

$$m\angle 8 = 110 \quad m\angle 4 = 110$$

$$m\angle 2 = 70 \quad m\angle 7 = 70$$



19. Relationships:

Angles 1 and 8: AEA \cong

Angles 1 and 5: Corr. \cong

Angles 4 and 8: Corr \cong

Angles 3 and 6: AIA \cong

Angles 7 and 6: VA \cong

Angles 7 and 8: LP $= 180$

Angles 3 and 5: SSI $= 180$

Angles 2 and 8: SSA $= 180$

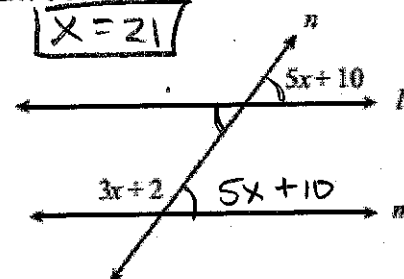
20. If $\angle 6 = 82$ and $\angle 3 = 2x + 10$, find x and angles 6 and 3.

$$x = 36 \quad m\angle 6 = 82$$

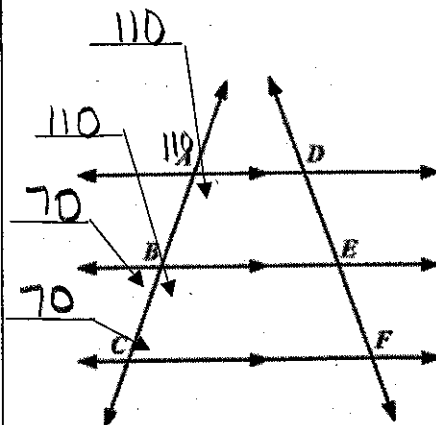
$$m\angle 3 = 82$$

21. Solve for x .

$$x = 21$$



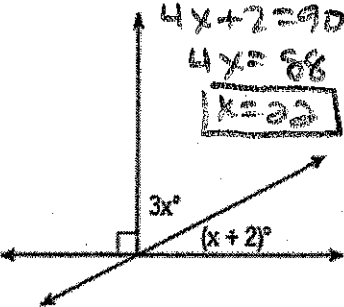
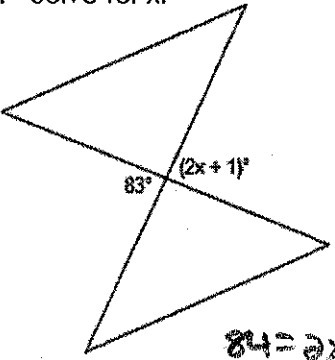
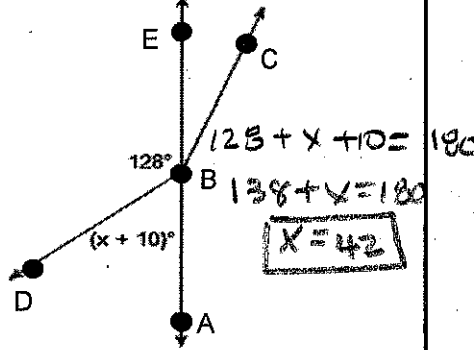
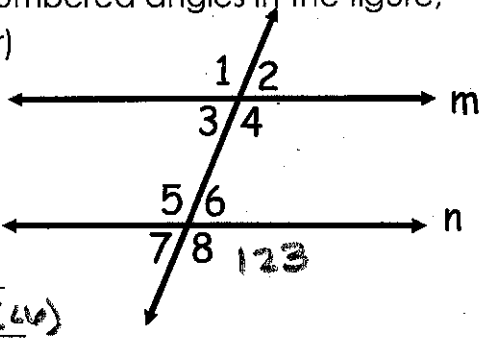
22. a. If $\angle A = 110$, find the angles the arrows are pointing to.



Name: _____

Date: _____

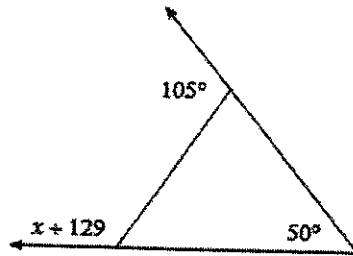
Use the following to review for you test. **Show your work on a separate sheet of paper if needed.**

Things to Know	Things to Remember	Examples
Solving for missing angles	Linear Pair - ___ + ___ = 180° Supplementary Angles ___ + ___ = 180° Complementary Angles ___ + ___ = 90° Vertical Angles ___ = ___ Alternate Interior Angles ___ = ___ Alternate Exterior Angles ___ = ___ Corresponding Angles ___ = ___ Consecutive Interior Angles ___ + ___ = 180°	<p>1. Solve for x.</p> $3x + x + 2 = 90$ $4x + 2 = 90$ $4x = 88$ $x = 22$  <p>2. Solve for x.</p>  <p>3. Solve for x, and the measure of $\angle ABD$</p>  <p>4. One of two supplementary angles is 98° greater than its supplement. Find the measure of both angles.</p> $m\angle 1 = x + 98$ $2x + 98 = 180$ $2x = 82$ $x = 41$ <p>5. $\angle 1$ and $\angle 2$ are complementary angles. Solve for x and the measure of both angles.</p> $\angle 1 = 7x + 20$ $\angle 2 = 17x - 2$ $24x + 18 = 90$ $24x = 72$ $x = 3$
<p>6. Given $m \parallel n$, $m\angle 8 = 123^\circ$, find the measures of all the numbered angles in the figure, and give the reason why (vocab in things to remember)</p>		
<p>$m\angle 1 = 123$, $m\angle 2 = 57$, $m\angle 3 = 57$ <u>VA \cong</u> <u>Corr \cong</u> <u>VA \cong</u> (1,6) (2,2)</p> <p>$m\angle 4 = 123$, $m\angle 5 = 123$, $m\angle 6 = 57$, $m\angle 7 = 57$ <u>VA \cong</u> <u>VA \cong</u> <u>Linear Pair</u> <u>VA \cong</u> (6,6)</p>		

Sum of Interior & Exterior Angles

The sum of all interior angles is 180° .
 $\angle 1 + \angle 2 + \angle 3 = 180^\circ$
 The sum of a straight line is 180° .

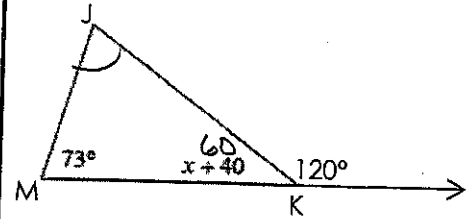
7. Solve for $x = -4$



$$x + 129 = 50 + 75$$

$$x = -4$$

8. Solve for $x = 20$ and $\angle J = 47$



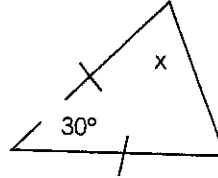
$$x + 40 = 60$$

$$x = 20$$

Base Angles

-If 2 angles in a triangle are congruent, then the sides opposite them are congruent.
 -If 2 sides in a triangle are congruent, then the angles opposite them are congruent.

11. Solve for x .

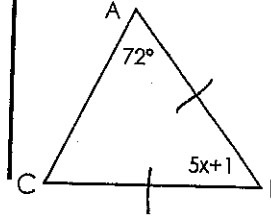


$$30 + 2x = 180$$

$$2x = 150$$

$$x = 75$$

12. $\triangle ABC$ is an isosceles triangle with AB and BC as the legs. Solve for x .



$$x = 7$$

$$5x + 1 + 72 + 72 = 180$$

$$5x + 145 = 180$$

$$5x = 35$$

$$x = 7$$