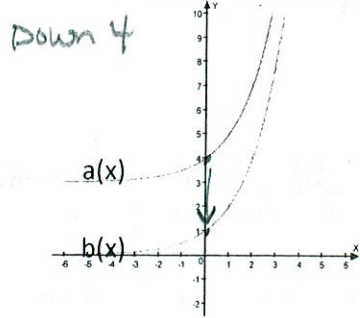


Unit 4 Exponential Functions Review

| Skill                                                                      | Things to remember                                                                                                                                                                                                                                      | Examples                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                 |    |     |    |   |   |   |      |     |   |    |     |                                                                                                                                                                                    |
|----------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|-----|----|---|---|---|------|-----|---|----|-----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. Determine if representations are exponential. Explain why or why not    | <p>Exponential Functions:<br/>-Variable in exponent<br/>-Constant Ratios<br/>-Graph is a curve</p> <p>Linear Functions:<br/>-Constant differences<br/>-Graph is a line</p>                                                                              | <p>a. Determine if the points are exponential or linear:</p> <table border="1"> <tr> <td>x</td> <td>-3</td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> </tr> <tr> <td>y</td> <td>0.16</td> <td>0.8</td> <td>4</td> <td>20</td> <td>100</td> </tr> </table> <p>EXP</p> <p>b. (-2, 5) (-1, 4) (0, 3) (1, 2) (2, 1) Linear</p> | x                                                                                                                                                                               | -3 | -2  | -1 | 0 | 1 | y | 0.16 | 0.8 | 4 | 20 | 100 | <p>b. Determine if the equations are linear or exponential:</p> <p>a. <math>y = 3^x - 4</math> EXP</p> <p>b. <math>y = 2^2</math> Linear</p> <p>c. <math>y = 6^{2x}</math> EXP</p> |
| x                                                                          | -3                                                                                                                                                                                                                                                      | -2                                                                                                                                                                                                                                                                                                                             | -1                                                                                                                                                                              | 0  | 1   |    |   |   |   |      |     |   |    |     |                                                                                                                                                                                    |
| y                                                                          | 0.16                                                                                                                                                                                                                                                    | 0.8                                                                                                                                                                                                                                                                                                                            | 4                                                                                                                                                                               | 20 | 100 |    |   |   |   |      |     |   |    |     |                                                                                                                                                                                    |
| 2. Determine if a function is exponential growth or decay and explain why. | <p><math>0 &lt; b &lt; 1</math>: Decay<br/><math>b &gt; 1</math>: Growth</p>                                                                                                                                                                            | <p>a. <math>y = .75\left(\frac{3}{2}\right)^x</math></p> <p>Growth <math>\frac{3}{2} = 1.5</math> greater than 1</p>                                                                                                                                                                                                           | <p>b. <math>y = \left(\frac{1}{2}\right)^x</math></p> <p>Decay <math>\frac{1}{2}</math> is less than 1</p>                                                                      |    |     |    |   |   |   |      |     |   |    |     |                                                                                                                                                                                    |
|                                                                            |                                                                                                                                                                                                                                                         | <p>c. What is the function growing by?<br/><math>Y = 3(2)^x</math></p> <p>2</p>                                                                                                                                                                                                                                                | <p>d. What is constant ratio?<br/><math>Y = 3(4.5)^x</math></p> <p>4.5</p>                                                                                                      |    |     |    |   |   |   |      |     |   |    |     |                                                                                                                                                                                    |
| 3. Graph an exponential function.                                          | <p><math>y = ab^x</math></p> <p>Create a table with values for x of -2, -1, 0, 1, and 2 (you may have to include other x-values if needed)</p>                                                                                                          | <p>a. Graph: <math>f(x) = \left(\frac{1}{2}\right)^x</math></p>                                                                                                                                                                                                                                                                | <p>b. Graph: <math>f(x) = 3 \cdot 2^x</math></p>                                                                                                                                |    |     |    |   |   |   |      |     |   |    |     |                                                                                                                                                                                    |
| 4. Describe the transformations of an exponential function.                | <p><math>f(x) = a(b)^{x-h} + k</math></p> <p><b>a</b> stretches or shrinks AND reflects</p> <p><b>k</b> moves the function up and down.</p> <p><b>h</b> moves the function left and right.</p> <p>The new asymptote is the line <math>y = k</math>.</p> | <p>a. Given the function <math>f(x) = 2^x</math> write a new equation after a transformation of left 7 and up 3.</p> <p><math>f(x) = 2^{x+7} + 3</math></p>                                                                                                                                                                    | <p>b. Given the function <math>g(x) = 2^x</math>, write a new equation after a transformation of right 9 and reflect across the x-axis.</p> <p><math>f(x) = -2^{x-9}</math></p> |    |     |    |   |   |   |      |     |   |    |     |                                                                                                                                                                                    |
|                                                                            |                                                                                                                                                                                                                                                         | <p>c. Describe the transformation <math>h(x) = 10^x</math> to <math>k(x) = 4(10)^{x+1} - 5</math>.</p> <p>Stretch 4<br/>Left 1<br/>Down 5</p>                                                                                                                                                                                  | <p>d. Describe the transformation from <math>a(x)</math> to <math>b(x)</math>.</p>                                                                                              |    |     |    |   |   |   |      |     |   |    |     |                                                                                                                                                                                    |



5. Create equations from a graph or table

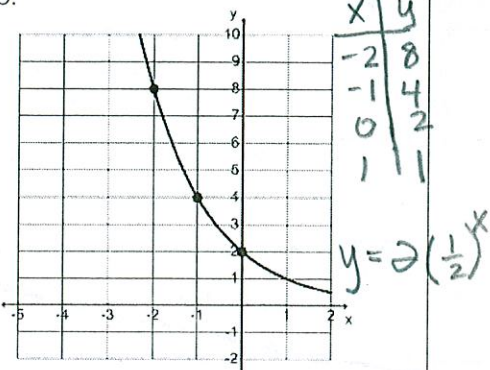
$y = y\text{-int}(\text{constant ratio})^x$

a.

|   |      |     |   |   |    |    |
|---|------|-----|---|---|----|----|
| x | 0    | 1   | 2 | 3 | 4  | 5  |
| y | 1/16 | 1/4 | 1 | 4 | 16 | 64 |

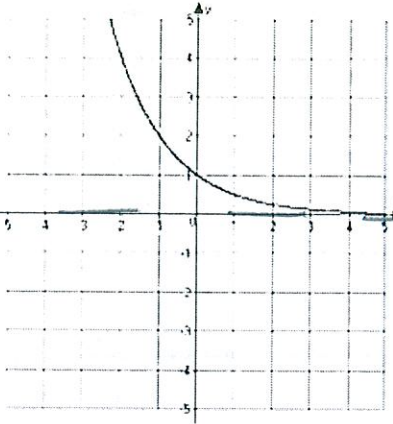
$y = a(b)^x$   
 ↑ y-int ↑ rate  
 $y = \frac{1}{16}(4)^x$

b.



6. Determine characteristics of exponential functions.

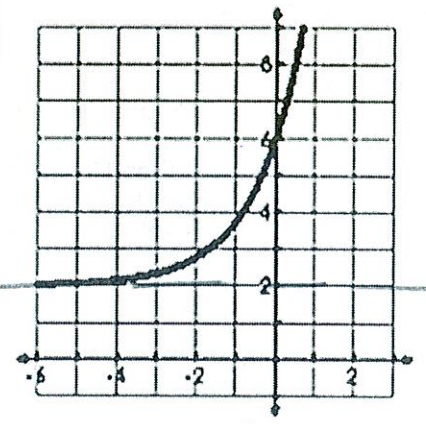
a.



$(-1, 4)$   
 $(0, 1)$   
 $\frac{4-1}{0+1} = \frac{3}{1}$   
 $(-2, 4)$   
 $(0, 1)$   
 $\frac{1-4}{0+2} = \frac{-3}{2}$

Domain:  $(-\infty, \infty)$   
 Range:  $(0, \infty)$   
 x-Intercept: None  
 y-intercept:  $(0, 1)$   
 Interval of Increase:  
 Interval of Decrease:  $(-\infty, \infty)$   
 Asymptote:  $y = 0$   
 End Behavior:  
 as  $x \rightarrow -\infty, f(x) \rightarrow \infty$   
 as  $x \rightarrow \infty, f(x) \rightarrow 0$   
 ROC from -2 to 0: -3 | 2

b.



Domain:  $(-\infty, \infty)$   
 Range:  $(2, \infty)$   
 x-Intercept: None  
 y-intercept:  $(0, 2)$   
 Interval of Increase:  $(-\infty, \infty)$   
 Interval of Decrease:  
 Asymptote:  $y = 2$   
 End Behavior:  
 as  $x \rightarrow -\infty, f(x) \rightarrow 2$   
 as  $x \rightarrow \infty, f(x) \rightarrow \infty$   
 ROC from -1 to 0: 2

7. Determine the y-intercept and asymptote from an equation

You can always substitute 0 in for x to find a y-intercept  
 Asymptote:  $y = k$   
 No 'k' value, the asymptote is  $y = 0$ .

a. Determine the y-intercept and asymptote of the function  $y = 3(2)^x$ .  
 $y: (0, 3)$   
 Asy:  $y = 0$

b. Determine the y-intercept and asymptote of the function  $y = 4(\frac{1}{2})^x - 2$ .  
 $y: (0, 2)$   
 Asy:  $y = -2$

| 8. Determine the growth/decay factor and percent. | $(1+r)$ and $(1-r)$ represent the growth and decay factors                                                               | <p>a. <math>y = 3(1.25)^x</math></p> <p>Determine if the function is <u>growth</u> or decay:</p> <p>Factor: <math>1.25</math></p> <p>Percent: <math>.25</math> or <math>25\%</math></p>                                                                                                                                                                                                                                                                                                                                                                                              | <p>b. <math>y = 2(.84)^x</math></p> <p>Determine if the function is <u>growth</u> or <u>decay</u>:</p> <p>Factor: <math>.84</math></p> <p>Percent: <math>.16</math> or <math>16\%</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |     |         |   |   |   |   |   |    |   |   |
|---------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|---------|---|---|---|---|---|----|---|---|
| 9. Applications of exponential functions.         | <p>Growth and decay</p> $y = a(1+r)^t$ $y = a(1-r)^t$ <p>Compound interest:</p> $A = P\left(1 + \frac{r}{n}\right)^{nt}$ | <p>a. Luke Duke deposits \$2000 into a bank account that pays 5% interest <u>compounded monthly</u>. Find the balance in the account after 4 years.</p> <p>Model: <math>y = 2000\left(1 + \frac{.05}{12}\right)^{12 \cdot 4}</math></p> <p>Solution: <u>\$2441.79</u></p> <p>c. A certain radioactive element <u>decays</u> at a rate of 21% <u>per month</u>. If the starting amount was 32 ounces, how much will be left after <u>1 year</u>?</p> <p>Model: <math>y = 32(1 - .21)^t</math></p> <p><math>t = 12</math> months</p> <p>Solution: <u>1.89 oz</u><br/><u>≈ 2 oz</u></p> | <p>b. The value of the Barbie Dream House is \$125,000. This house is in a prime location and <u>appreciates</u> (increases in value) at a rate of 7% <u>per year</u>. How much will the Barbie Dream House be worth in 5 years?</p> <p>Model: <math>y = 125000(1 + .07)^5</math></p> <p>Solution: <u>\$175,318.97</u></p> <p>d. Michael is offered two jobs – Job A, which offers him a starting salary of \$20,000 a year with a 5% raise each year he works there and Job B, which offers him a starting salary of \$25,000, but only a 3% raise each year. Michael plans to work to work at the job for 7 years. Which job should he pick and why?</p> <p>SKIP</p> |     |         |   |   |   |   |   |    |   |   |
| 10. Geometric sequences                           | <p>Explicit formula</p> $a_n = a_1 r^{n-1}$ $a_n = r(a_{n-1})$                                                           | <p>a. Given 3, 6, 12, ... find:</p> <p>Explicit formula:</p> $a_n = 3(2)^{n-1}$ <p>9<sup>th</sup> term: <u>768</u></p> <p>Recursive formula:</p> $a_1 = 3$ $a_n = 2 \cdot a_{n-1}$                                                                                                                                                                                                                                                                                                                                                                                                   | <p>b. Day 1, Mrs. Haggard found 2 pencils on the ground. Day 2, her collection had grown to 6 pencils. By day 3, she has 18 pencils.</p> <p>Explicit formula:</p> $a_n = 2(3)^{n-1}$ <p>How many pencils did she have on day 7?</p> <p><u>1458 pencils</u></p> <p>Recursive formula:</p> $a_1 = 2$ $a_n = 3 \cdot a_{n-1}$                                                                                                                                                                                                                                                                                                                                             |     |         |   |   |   |   |   |    |   |   |
|                                                   |                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | <table border="1"> <thead> <tr> <th>Day</th> <th>pencils</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>2</td> </tr> <tr> <td>2</td> <td>6</td> </tr> <tr> <td>3</td> <td>18</td> </tr> <tr> <td>⋮</td> <td>⋮</td> </tr> </tbody> </table>                                                                                                                                                                                                                                                                                                                                                                                                                            | Day | pencils | 1 | 2 | 2 | 6 | 3 | 18 | ⋮ | ⋮ |
| Day                                               | pencils                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |     |         |   |   |   |   |   |    |   |   |
| 1                                                 | 2                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |     |         |   |   |   |   |   |    |   |   |
| 2                                                 | 6                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |     |         |   |   |   |   |   |    |   |   |
| 3                                                 | 18                                                                                                                       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |     |         |   |   |   |   |   |    |   |   |
| ⋮                                                 | ⋮                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |     |         |   |   |   |   |   |    |   |   |

|                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                                                                                                                                                        |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>11. Exponent rules</p> <p><math>(x^a)(x^b) = x^{a+b}</math></p> <p><math>(x^a)^b = x^{ab}</math></p> <p><math>\frac{x^a}{x^b} = x^{a-b}</math></p> <p><math>x^0 = 1 \quad x^{-a} = \frac{1}{x^a}</math></p> | <p>a. <math>(-3xy^3)(2x^2y^4)^2</math></p> <p><math>-3xy^3 \cdot 2^2 x^4 y^8</math></p> <p><math>-12x^5 y^{11}</math></p> <p>c. <math>\frac{-10x^2 y^3 z^2}{5x^6 y^7 z^6}</math></p> <p><math>\frac{-10x^2 z^2 z^6}{5x^2 y^7 y^5} = \frac{-2x^4 z^8}{y^{12}}</math></p> <p>f. <math>\left( \frac{(-2x^{-3}y^{-4})^2 (3x^4y^{-5})^{-3}}{(xy)^3 (4x^{-1}y^2)} \right)^{-2}</math></p> | <p>d. <math>\frac{4x^3 y^5}{(2x^6 y^2)^3}</math></p> <p><math>\frac{4x^3 y^5}{2^3 x^{18} y^6} = \frac{4}{8x^{15} y} = \frac{1}{2x^{15} y}</math></p> <p>e. <math>\frac{(3x^{-1}y^3z^0)^2}{(2x^3y^{-4}z)^{-2}}</math></p> <p><math>\frac{3^2 x^{-2} y^6 z^0}{2^{-2} x^{-6} y^8 z^{-8}}</math></p> <p><math>\frac{3^2 2^2 x^6 y^6 z^0 z^8}{x^2 y^8}</math></p> <p><math>\frac{36x^4 z^8}{y^2}</math></p> |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

$$\left[ \frac{-2^2 x^{-6} y^{-8} \cdot 3^{-3} x^{-12} y^{15}}{x^3 y^3 \cdot 4x^{-1} y^2} \right]^{-2}$$

$$\left[ \frac{-2^2 x^{-18} y^7 \cdot 3^{-3}}{4x^2 y^5} \right]^{-2}$$

$$\frac{-2^{-4} x^{36} y^{-14} \cdot 3^6}{4^{-2} x^{-4} y^{-10}}$$

$$\frac{3^6 x^{36} \cdot 4^2 x^4 y^{10}}{-2^4 \cdot y^{14}}$$

$$\frac{729 x^{40} y^{10} \cdot 16}{-16 y^{14}}$$

$$\frac{-729 x^{40}}{y^4}$$