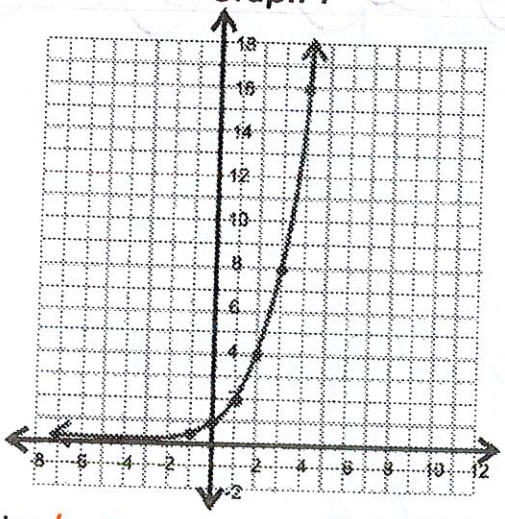
Day Three: Characteristics of Exponentials

One key component to fully understanding quadratic functions is to be able to describe characteristics of the graph and its equation.

Domain and Range		
Domain		
Define: All possible values of x	**Think:** How far left to right does the graph go?	**Write:** All real #s $\mathbb{R}$ $(-\infty, \infty)$
Range		
Define: All possible values of y	**Think:** How far down to how far up does the graph go?	**Write:** $(Asy., \infty)$ $(-\infty, Asy.)$

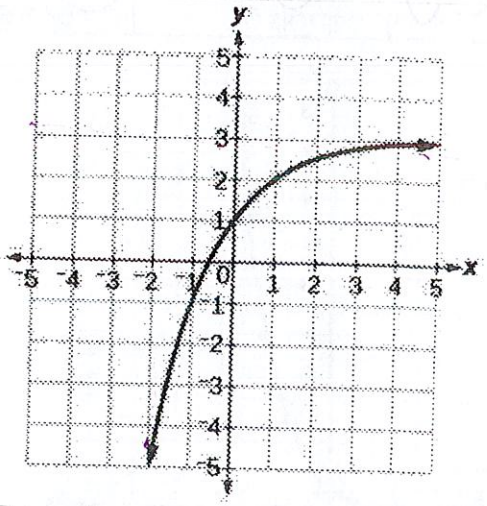
Graph 1



Domain: $(-\infty, \infty)$

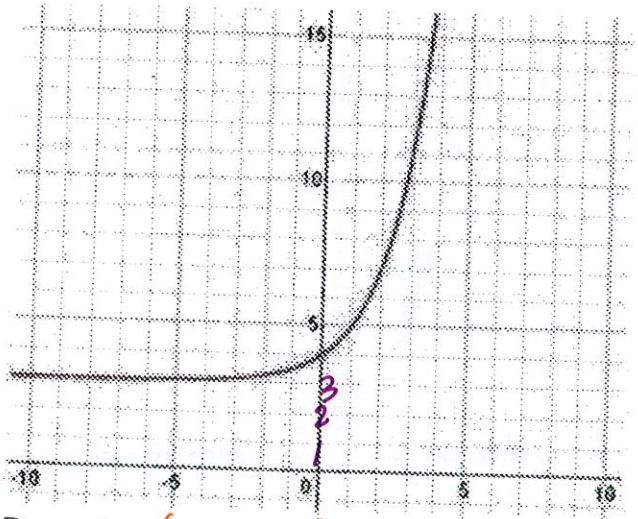
Range: $(0, \infty)$

Graph 2



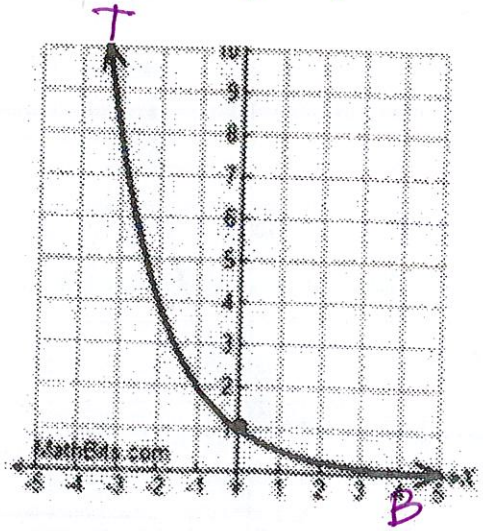
Domain: $(-\infty, \infty)$

Range: $(-\infty, 3)$



Domain: $(-\infty, \infty)$

Range: $(3, \infty)$



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Zeros and Intercepts

Y-Intercept

Define:	Think:	Write:
Point where the graph crosses the y-axis	At what coordinate point does the graph cross the y-axis?	(0, y)

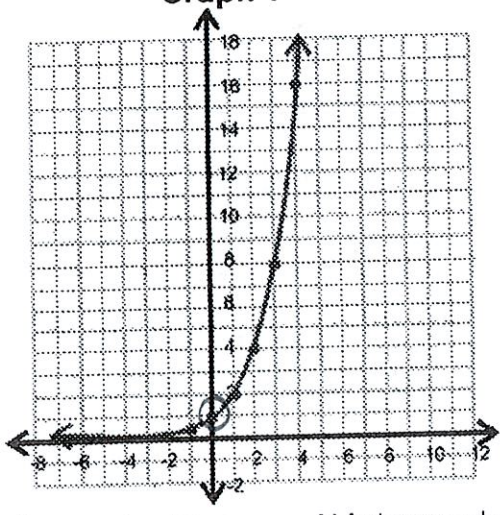
X-Intercept

Define:	Think:	Write:
Point where the graph crosses the x-axis	At what coordinate point does the graph cross the x-axis?	(x, 0)

Zero

Define:	Think:	Write:
Where the function (y-value) equals 0	At what x-value does the graph cross the x-axis?	

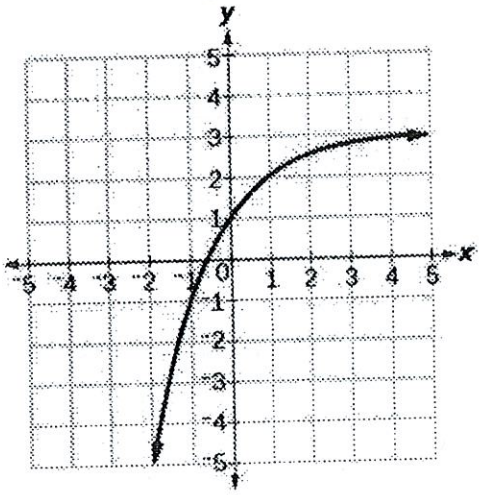
Graph 1



X-intercepts: None Y-intercept: (0, 1)

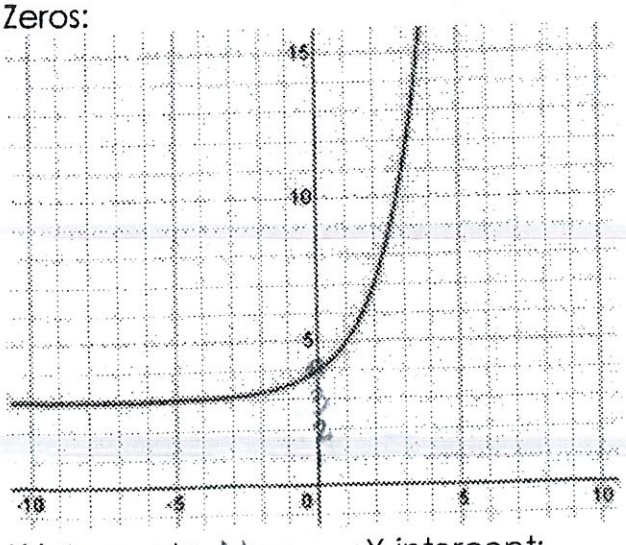
Zeros:

Graph 2



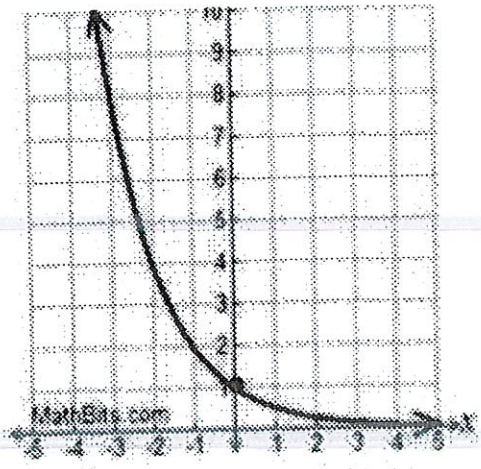
X-intercepts: (-1/2, 0) Y-intercept: (0, 1)

Zeros:



X-intercepts: None Y-intercept: (0, 4)

Zeros:



X-intercepts: None Y-intercept: (0, 1)

Zeros:

Extrema and Asymptote

Maximum

Define:	Think:	Write:
Highest point or peak of a function.	What is my highest point on my graph?	

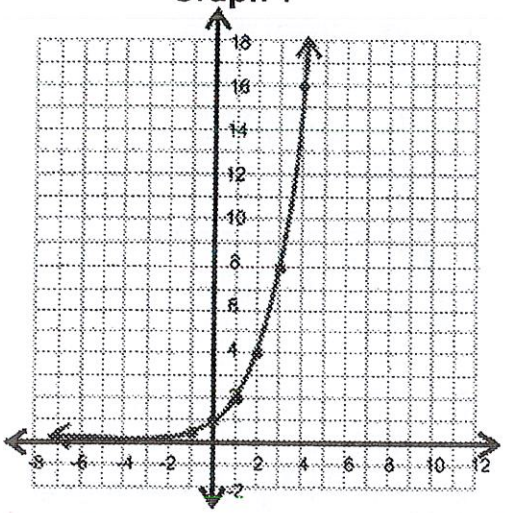
Minimum

Define:	Think:	Write:
Lowest point or valley of a function.	What is the lowest point on my graph?	

Asymptote

Define:	Think:	Write:
A line that the graph gets closer and closer to, but never touches or crosses	Where could I draw a horizontal line that is a boundary?	$y = k$

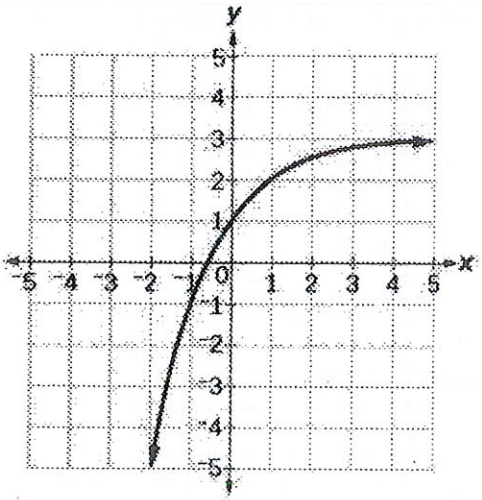
Graph 1



Minimum: Maximum:

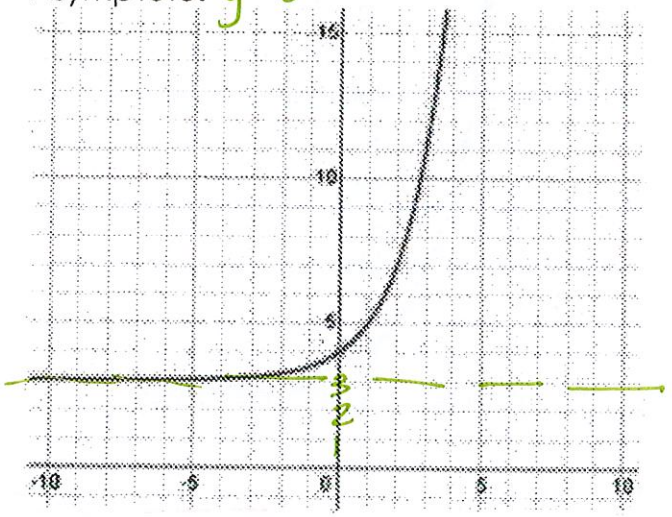
Asymptote: $y = 0$

Graph 2



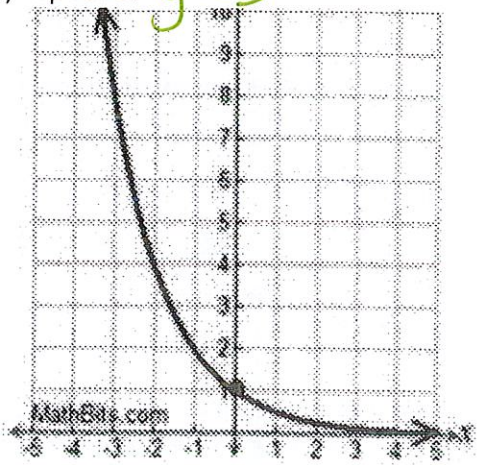
Minimum: Maximum:

Asymptote: $y = 3$



Minimum: Maximum:

Asymptote: $y = 3$



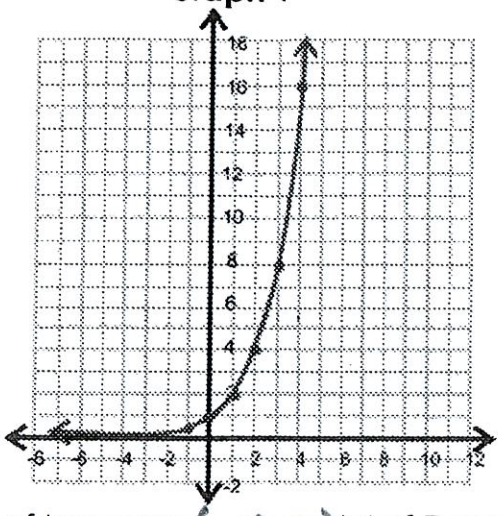
Minimum: Maximum:

Asymptote: $y = 0$

Intervals of Increase, Decrease, and Constant

Interval of Increase		
Define: The part of the graph that is rising as you read left to right.	**Think:** From left to right, is my graph going up?	**Write:** Positive slope $(-\infty, \infty)$
Interval of Decrease		X Values
Define: The part of the graph that is falling as you read from left to right.	**Think:** From left to right, is my graph going down?	**Write:** Negative slope $(-\infty, \infty)$
Interval of Constant		
Think: The part off the graph that is a horizontal line.	**Think:** From left to right, is my graph flat?	**Think:**

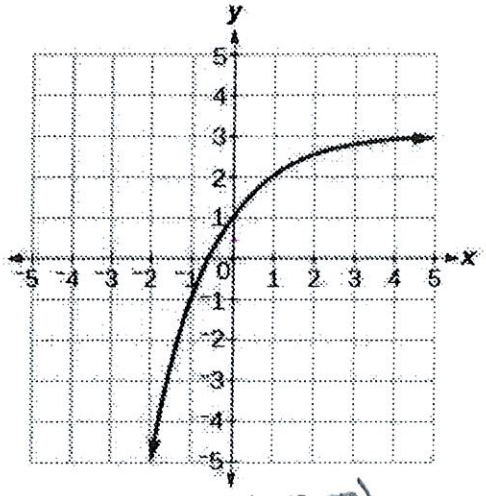
Graph 1



Int of Increase: $(-\infty, \infty)$ Int of Decrease: X

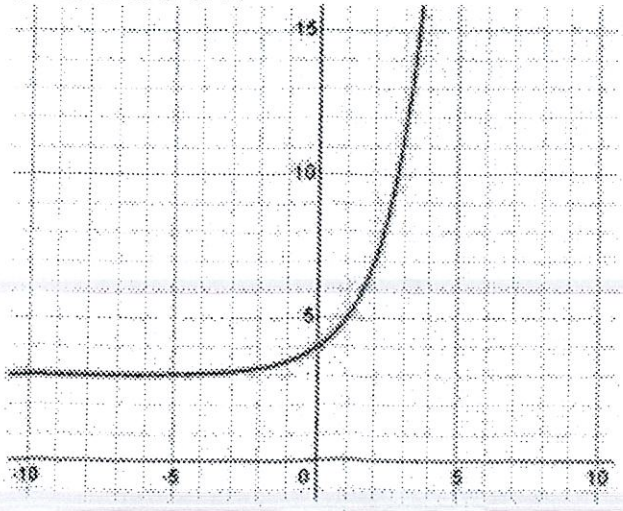
Int of Constant: None

Graph 2



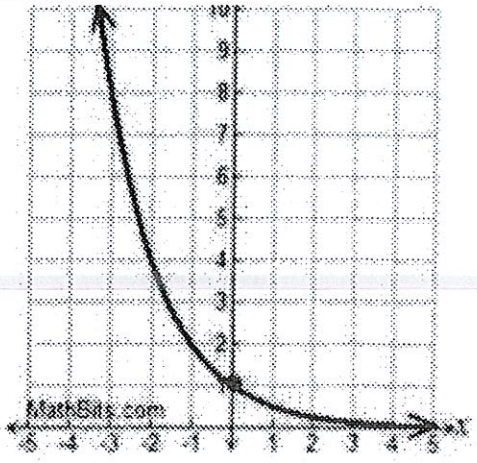
Int of Increase: $(-\infty, \infty)$ Int of Decrease: X

Int of Constant:



Int of Increase: $(-\infty, \infty)$ Int of Decrease: X

Int of Constant:



Int of Increase: X Int of Decrease: $(-\infty, \infty)$

Int of Constant:

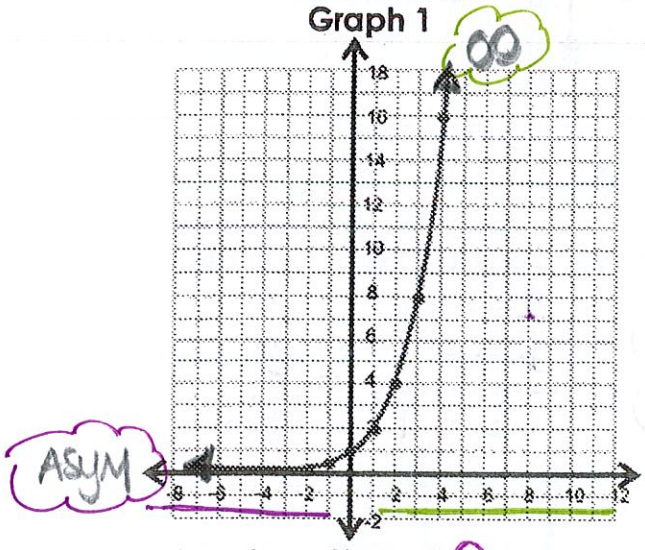
End Behavior

End Behavior

Define:
Behavior of the ends of the function (what happens to the y-values or f(x)) as x approaches positive or negative infinity. The arrows indicate the function goes on forever so we want to know where those ends go.

Think:	**Write:**
As x goes to the left (negative infinity), what direction does the left arrow go?	As x → -∞, f(x) → ∞ / -∞ (Asymptote)
Think: As x goes to the right (positive infinity), what direction does the right arrow go?	**Write:** As x → ∞, f(x) → ∞ / -∞ (Asymptote)

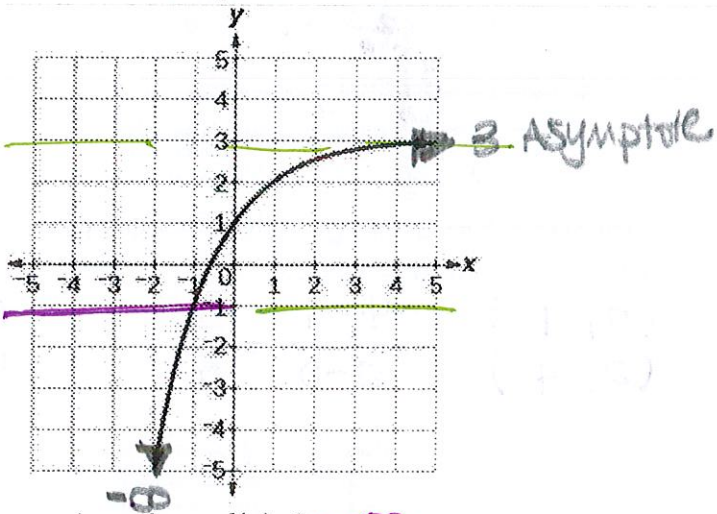
Graph 1



As x → -∞, f(x) → 0.

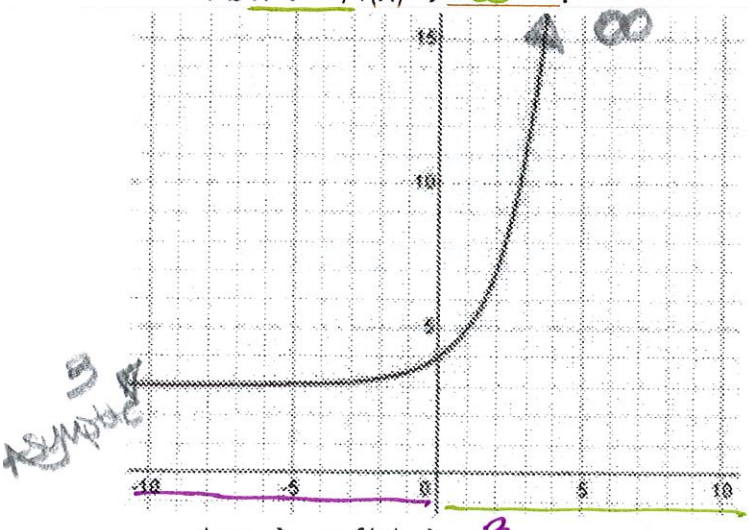
As x → ∞, f(x) → ∞.

Graph 2



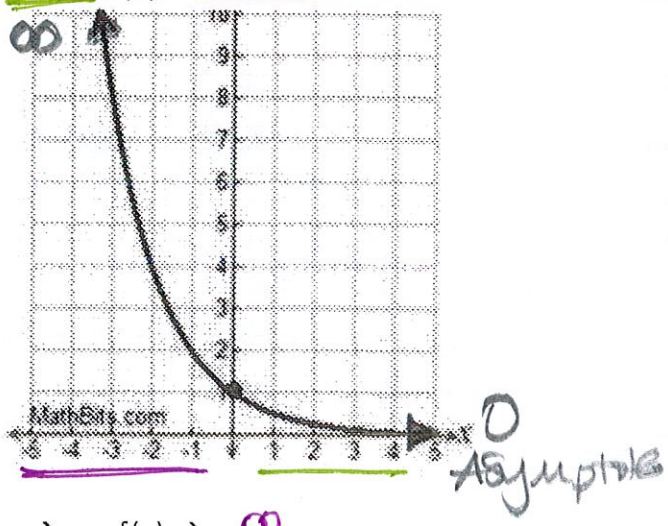
As x → -∞, f(x) → -∞.

As x → ∞, f(x) → 3.



As x → -∞, f(x) → 3.

As x → ∞, f(x) → ∞.



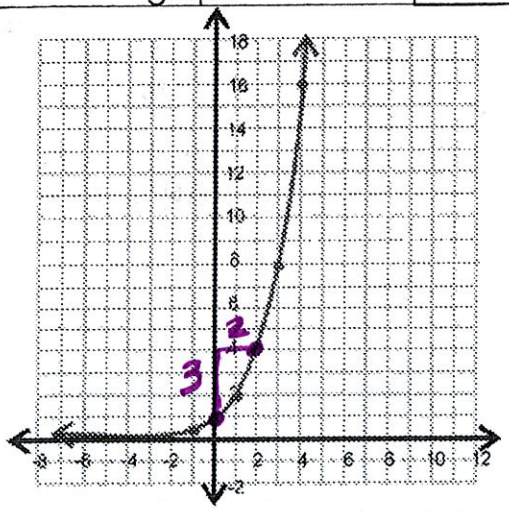
As x → -∞, f(x) → ∞.

As x → ∞, f(x) → 0.

Average Rate of Change

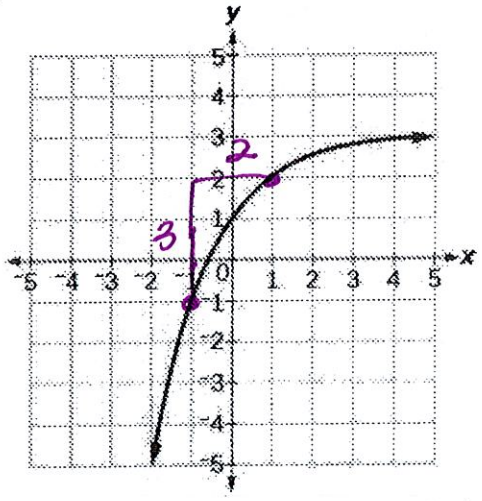
Average Rate of Change

Define:	Think:	Write:
Rate of change or slope for a given interval on a graph	How is the graph changing over the given interval?	$AROC = \frac{y_2 - y_1}{x_2 - x_a}$ Slope =



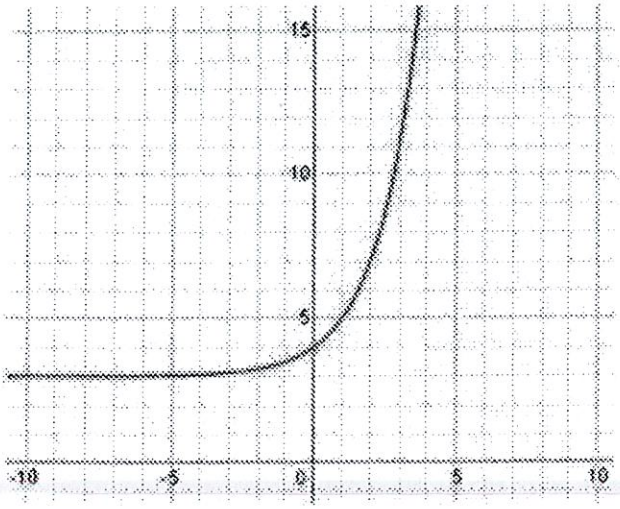
Calculate the average rate of change for the interval $0 \le x \le 2$.

(0, 1) (2, 4) $\frac{4-1}{2-0} = \frac{3}{2}$

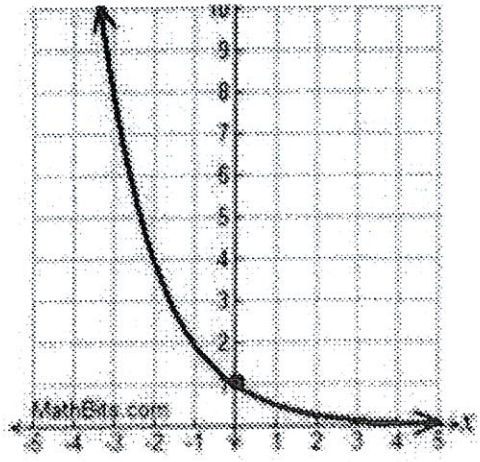


Calculate the average rate of change for the interval $-2 \le x \le 1$.

(-1, -1) (1, 2) $\frac{2-(-1)}{1-(-1)} = \frac{3}{2}$



Calculate the average rate of change for the interval $0 \le x \le 3$.



Calculate the average rate of change for the interval $-3 \le x \le 0$.

Calculate the average rate of change for the function $f(x) = 3^x$ for the interval $1 \le x \le 3$.

(1, 3) (3, 27) $\frac{27-3}{3-1} = \frac{24}{2} = 12$

$3 \le x \le 5$

(3, 27) (5, 243) $\frac{243-27}{5-3} = \frac{216}{2} = 108$

Growth: $P(1+\frac{r}{n})^{nt}$

1) Given the equation $y = 15(1.75)^x$
   a) Does this equation represent growth or decay? G
   b) What is the rate of growth or decay? .75
   c) What is the initial value? 15
   d) Evaluate for $x = 4$ 140.68

2) Given the equation $y = 25(1.23)^x$
   a) Does this equation represent growth or decay? G
   b) What is the rate of growth or decay? .23
   c) What is the initial value? 25
   d) Evaluate for $x = 2$ 37.82

3) Given the equation $y = 154(1.06)^x$
   a) Does this equation represent growth or decay? G
   b) What is the rate of growth or decay? .06
   c) What is the initial value? 154
   d) Evaluate for $x = 7$ 231.56

4) The tuition at a private college was $15,000. The tuition has about a 7.2% increase.
   a) Write an exponential equation describing this situation. $15000(1+.072)^t$
   b) How much will the tuition be 5 years from now? $\$21,235.63$

5) A vintage radio was purchased by Grandma Schmidt in 1945 for $16. You radio know what it's worth. Each year the radio's value increased 1.4%.
   a) Write an exponential equation describing this situation. $16(1+0.014)^t$
   b) What will the radio be worth in 2020? 75 year = time $\$45.39$

6) Ben made $2,000 last summer mowing lawns. He wants to continue this b following years. He hopes that his profit will increase 10%.
   a) Write an exponential equation describing the situation. $2000(1+.10)^t$
   b) What will he make in 7 years? $\$3,897.43$

7) Marcus collects baseball cards. He paid $250 for a Babe Ruth collectors card in 2008. Each year, the cards value increases by 3%

a) Write an exponential equation describing the situation. $250(1+.03)^t$

b) What will it be worth in 10 years? $\$335.98$

8) The population of an ant farm increases at a rate of 20.6% per day.

a) If we start with 56 ants, can you find a formula that models this rate of growth? $56(1+.206)^t$

b) How many will be on the farm at the end of day 4? $118.46 \rightarrow 119$ (118)

c) Will there be more than 1,000 ants after a week? About how many days does it take to populate more than 5,000 ants? 7 — Solve for time

7 day: $207.78 \approx 208$ No! → 24 days

9) You just won the lotto. You have two options, you can take the full $5,000,000 now or they will pay you 5,000 dollars this year and then double the amount they pay you every year for 10 years. Which way will get you more money? → $y = a(b)^x$

① 5,000,000 * ② $5{,}000(2)^{10} = \$5{,}120{,}000$