Proving a triangle is a right triangle

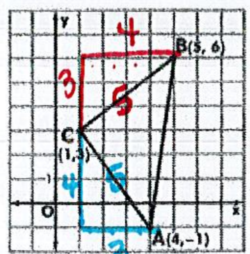
Method 1: Show two sides of the triangle are perpendicular by demonstrating their slopes are opposite reciprocals.

Method 2: Calculate the distances of all three sides and then test the Pythagorean's theorem to show the three lengths make the Pythagorean's theorem true.

Example 1:

Given: The triangle with vertices $A(4, -1)$, $B(5, 6)$, and $C(1, 3)$.

Show: $\triangle ABC$ is an isosceles right triangle.



Slope Formula

$\frac{y_2 - y_1}{x_2 - x_1}$

Slope: $BC = \frac{6-3}{5-1} = \frac{3}{4}$ } opp. recip → $\perp$

$CA = \frac{-1-3}{4-1} = -\frac{4}{3}$

Proving a Quadrilateral is a Parallelogram

Method 1: Show that the diagonals bisect each other by showing the midpoints of the diagonals are the same

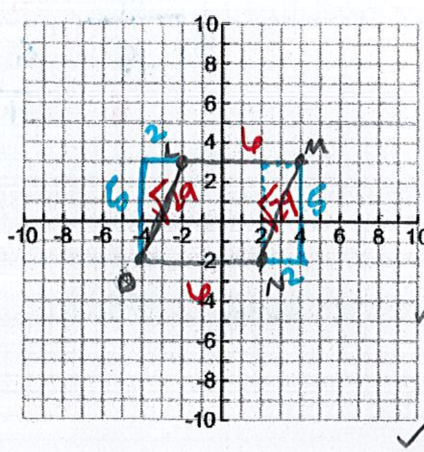
Method 2: Show both pairs of opposite sides are parallel by showing they have equal slopes.

Method 3: Show both pairs of opposite sides are equal by using distance.

Method 4: Show one pair of sides is both parallel and equal.

Examples

1. Prove that the quadrilateral with the coordinates L(-2,3), M(4,3), N(2,-2) and O(-4,-2) is a parallelogram.



$\boxed{\text{Method 1}}$: $\overline{LN}$

Mdpt $\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$ $\left(\frac{2-2}{2}, \frac{-2+3}{2}\right)$

$\overline{LN}$: $\left(0, \frac{1}{2}\right)$

$\overline{MO}$ $\left(\frac{-4+4}{2}, \frac{-2+3}{2}\right) = \left(0, \frac{1}{2}\right)$

$\boxed{\text{Method 2}}$: LM = ON slope zero

LO = MN Slope of $\frac{5}{2}$

$\boxed{\text{Method 3}}$: LM = ON Dist 6

LO = MN Dist $\sqrt{29}$

$\boxed{\text{Method 4}}$: Done is 2 & 3

Proving a Quadrilateral is a Rectangle

Prove that it is a parallelogram first, then:

Method 1: Show that the diagonals are congruent.

Method 2: Show that it has a right angle by using slope.

Examples:

1. Prove a quadrilateral with vertices G(1,1), H(5,3), I(4,5) and J(0,3) is a rectangle.

✓ $\boxed{\text{Method 1}}$

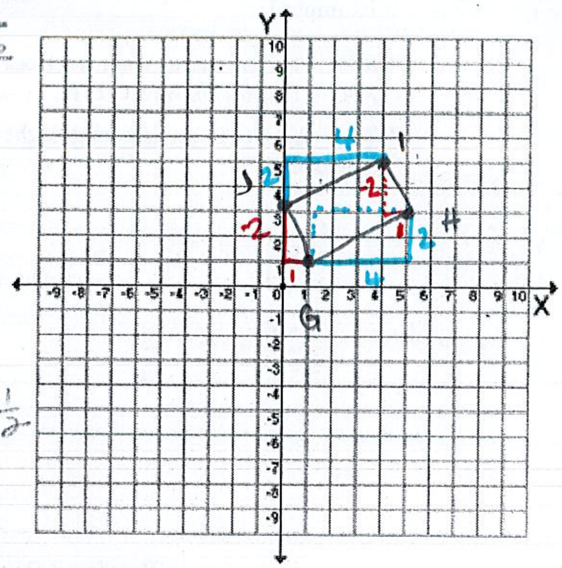
$\sqrt{(4-1)^2+(5-1)^2}$ $\qquad$ $\sqrt{(0-5)^2+(3-3)^2}$

$\sqrt{3^2+4^2}$ $\qquad$ $\sqrt{(-5)^2+0^2}$

$\sqrt{9+16}$ $\qquad$ $\sqrt{25}$

$\sqrt{25}$ $\qquad$ 5

5



$\boxed{\text{Method 2}}$ JG : -2 $\qquad$ IH $= -2$

JI : $\frac{2}{4}=\frac{1}{2}$ ✓ GH $= \frac{2}{4}=\frac{1}{2}$

Proving a Quadrilateral is a Rhombus

Prove that it is a parallelogram first, then:

Method 1: Prove that the diagonals are perpendicular.

Method 2: Prove that a pair of adjacent sides are equal.

Method 3: Prove that all four sides are equal.

Examples:

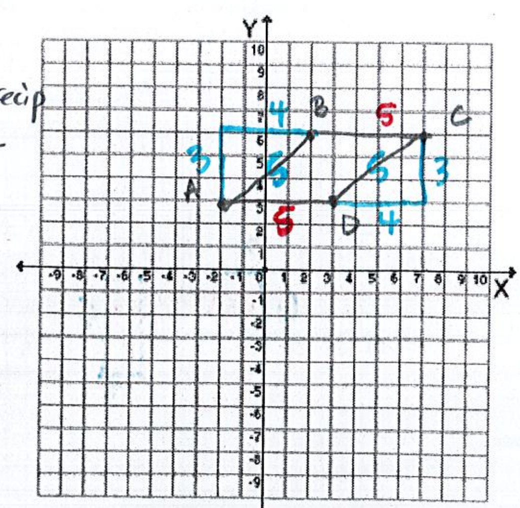
1. Prove that a quadrilateral with the vertices A(-2,3), B(2,6), C(7,6) and D(3,3) is a rhombus.

✓ $\boxed{\text{Method 1}}$

AC : $\frac{6-3}{7--2}=\frac{3}{9}=\frac{1}{3}$

BD $\frac{3-6}{3-2}=\frac{-3}{1}$

opp. recip
$\perp$



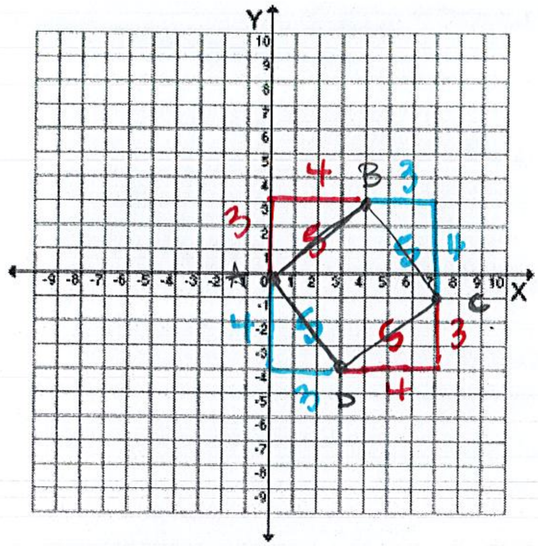
Proving that a Quadrilateral is a Square

There are many ways to do this. I recommend proving the diagonals bisect each other (parallelogram), are equal (rectangle) and perpendicular (rhombus).

Examples:

1. Prove that the quadrilateral with vertices A(0,0), B(4,3), C(7,-1) and D(3,-4) is a square.

* All side lengths are equal



* All right Angles

$AB = \frac{3}{4}$ $AD = -\frac{4}{3}$

$BC = -\frac{4}{3}$ $BC = \frac{3}{4}$

Proving a Quadrilateral is a Trapezoid

Show one pair of sides are parallel (same slope) and one pair of sides are not parallel (different slopes).

Proving a Quadrilateral is an Isosceles Trapezoid

Prove that it is a trapezoid first, then:

Method 1: Prove the diagonals are congruent using distance.

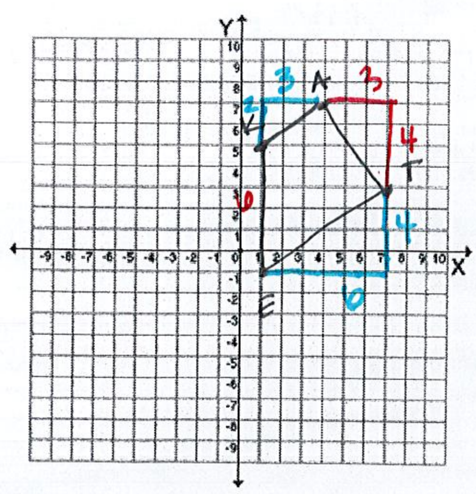
Method 2: Prove that the pair of non parallel sides are equal.

Examples:

1. Prove that KATE a trapezoid with coordinates K(1,5), A(4,7), T(7,3) and E(1,-1).

KE : undef slope

AT : $-\frac{4}{3}$



KA : $\frac{2}{3}$

ET : $\frac{4}{6} = \frac{2}{3}$