You may have discovered the following formula for conditional probability.

Conditional Probability

The conditional probability of B given A (the probability that event B occurs given that event A occurs) is given by the following formula:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

"B given A" [handwritten]

GIVEN ↑ [handwritten] total population [handwritten]

MCC9–12.S.CP3

3 EXAMPLE Using the Conditional Probability Formula

In a standard deck of playing cards, find the probability that a red card is a queen.

A Let event Q be the event that a card is a queen. Let event R be the event that a card is red. You are asked to find $P(Q \mid R)$. First find $P(R \cap Q)$ and $P(R)$.

$R \cap Q$ represents cards that are both red and a queen; that is, red queens.

There are ___2___ red queens in the deck of 52 cards, so $P(R \cap Q) =$ $\frac{2}{52}$.

There are ___26___ red cards in the deck, so $P(R) =$ $\frac{26}{52}$.

B Use the formula for conditional probability.

$$P(Q \mid R) = \frac{P(Q \cap R)}{P(R)} = \frac{\frac{2}{52}}{\frac{26}{52}}$$ Substitute probabilities from above.

$$= \frac{\ \ \ }{\ \ \ }$$ Multiply numerator and denominator by 52.

$$= \frac{2}{26}$$ Simplify.

So, the probability that a red card is a queen is $\frac{1}{13}$.

REFLECT

3a. How can you interpret the probability you calculated above?

_____

_____

3b. Is the probability that a red card is a queen equal to the probability that a queen is red? Explain.

_____

1. In order to study the connection between the amount of sleep a student gets and his or her school performance, data was collected about 120 students. The two-way table shows the number of students who passed and failed an exam and the number of students who got more or less than 6 hours of sleep the night before.

	Passed Exam	Failed Exam	TOTAL
Less than 6 hours of sleep	12	10	22
More than 6 hours of sleep	90	8	98
TOTAL	102	18	120

$P(<6 \mid F)$

a. To the nearest percent, what is the probability that a student who failed the exam got less than 6 hours of sleep? $\dfrac{10}{18}$

b. To the nearest percent, what is the probability that a student who got less than 6 hours of sleep failed the exam? _____

c. To the nearest percent, what is the probability that a student got less than 6 hours of sleep and failed the exam? _____

2. A botanist studied the effect of a new fertilizer by choosing 100 orchids and giving 70% of these plants the fertilizer. Of the plants that got the fertilizer, 40% produced flowers within a month. Of the plants that did not get the fertilizer, 10% produced flowers within a month. Find each probability to the nearest percent. (*Hint:* Construct a two-way table.)

	F	NF	
P	28	3	
PNP	42	27	

a. Find the probability that a plant that produced flowers got the fertilizer. _____

b. Find the probability that a plant that got the fertilizer produced flowers. _____

3. At a school fair, a box contains 24 yellow balls and 76 red balls. One-fourth of the balls of each color are labeled "Win a prize." Find each probability as a percent.

	B	WP	
Y	18	6	
R	57	19	
		25	

a. Find the probability that a ball labeled "Win a prize" is yellow. $\dfrac{6}{25}$

b. Find the probability that a ball labeled "Win a prize" is red. _____

c. Find the probability that a ball is labeled "Win a prize" and is red. _____

d. Find the probability that a yellow ball is labeled "Win a prize." _____

In Exercises 4–9, consider a standard deck of playing cards and the following events: A: the card is an ace; B: the card is black; C: the card is a club. Find each probability as a fraction.

4. $P(A \mid B)$ A given B
$\dfrac{2}{26} = \dfrac{1}{13}$

5. $P(B \mid A)$ B given A
$\dfrac{2}{4} = \dfrac{1}{2}$

6. $P(A \mid C)$ A given C
$\dfrac{1}{13}$

7. $P(C \mid A)$

8. $P(B \mid C)$

9. $P(C \mid B)$