

ay 3: Solve each equation by completing the square.

- * Not factorable
- * b term is even
- * Find the vertex (convert)

1) $r^2 - 4r - 21 = 0$

$$\boxed{r=7} \quad \boxed{r=-3}$$

* $r-2$.

$$\begin{array}{|c|c|} \hline r & r^2 \\ \hline -3 & \boxed{r^2} \\ \hline \end{array} \quad (r-2)(r-2) - 21 = 0 + 4$$

$$(r-2)^2 - 21 = 4$$

$$\sqrt{(r-2)^2} = \sqrt{25}$$

3) $x^2 + 16x - 57 = 0$

$$X = \frac{-b}{2a} \quad r-2 = \pm 5$$

$$(x+8)^2 - 121 = 0$$

* vertex form *

$$X = \frac{-16}{2(1)} \quad (x+8)^2 = 121$$

$$X = -8 \quad x+8 = \pm 11$$

y value of vertex

$$X = -8 \pm 11$$

$$\boxed{x=3} \quad \boxed{x=-19}$$

5) $a^2 - 16a + 22 = 0$

$$(a-8)^2 = -22 + 64$$

$$(a-8)^2 = 42$$

$$a-8 = \pm \sqrt{42}$$

$$a = 8 \pm \sqrt{42}$$

7) $x^2 + 4x - 32 = 0$

$$(x+2)^2 = 32 + 4$$

$$(x+2)^2 = 36$$

$$x+2 = \pm 6$$

$$x = -2 \pm 6$$

$$\boxed{x=4} \quad \boxed{x=-8}$$

9) $n^2 - 4n - 12 = 0$

$$(n-2)^2 = 12 + 4$$

$$(n-2)^2 = 16$$

$$n-2 = \pm 4$$

$$n = 2 \pm 4$$

$$\boxed{n=6} \quad \boxed{n=-2}$$

11) $8x^2 + 16x - 90 = 0$

$$8x^2 + 16x = 90$$

$$\boxed{x=0.5} \quad \boxed{x=-4.5}$$

$$\frac{8x^2 + 16x + 1}{(b/2)^2} = \frac{90}{8} + \frac{1}{8}$$

$$8(x+1)^2 = 98$$

$$(x+1)^2 = 12.25$$

2) $x^2 - 4x - 60 = 0$

$$x^2 - 4x + \frac{16}{4} = 60 + 4$$

$$(x-2)^2 = 64$$

$$4) b^2 + 6b + 8 = 0$$

$$(b+3)^2 = -8 + 9$$

$$(b+3)^2 = 1$$

$$(b+3) = \pm 1$$

$$b = -3 \pm 1$$

$$\boxed{b=-4} \quad \boxed{b=-2}$$

6) $n^2 + 16n - 80 = 0$

$$(n+8)^2 = 80 + 64$$

$$(n+8)^2 = 144$$

$$n+8 = \pm 12$$

$$n = -8 \pm 12$$

$$\boxed{n=4} \quad \boxed{n=-20}$$

8) $r^2 + 2r - 99 = 0$

$$(r+1)^2 = 99 + 1$$

$$(r+1)^2 = 100$$

$$r+1 = \pm 10$$

$$r = -1 \pm 10$$

$$\boxed{r=9} \quad \boxed{r=-11}$$

10) $k^2 - 12k + 32 = 0$

$$(k-4)^2 = 32 + 36$$

$$(k-4)^2 = 68$$

$$k-4 = \pm 2\sqrt{17}$$

$$\boxed{k=6 \pm 2\sqrt{17}}$$

$$x = \frac{-b}{2a}$$

12) $5b^2 + 10b - 45 = 0$

$$5(b^2 + 2b - 9) = 0$$

$$x = -1$$

$$5(x+1)^2 - 50 = 0$$

$$5(x+1)^2 = 50$$

$$(x+1)^2 = 10$$

$$\boxed{x=-1 \pm \sqrt{10}}$$

$$x+1 = \pm \sqrt{10}$$

$$13) 9x^2 - 18x - 55 = 0$$

$$9x^2 - 18x = 55$$

$$X = 1 \pm \sqrt{7.1}$$

$$14) 3x^2 - 18x - 21 = 0$$

$$X = 7 \quad X = -1$$

$$21) x^2 - 2x - 55 = 0$$

$$9(x-1)^2 = 55 + 9$$

$$9(x-1)^2 = 64$$

$$(x-1)^2 = 7.1$$

$$15) 4x^2 - 16x + 15 = 0$$

$$4(x^2 - 4x) = -15$$

$$4(x-2)^2 = -15 + 16$$

$$4(x-2)^2 = 1$$

$$(x-2)^2 = \frac{1}{4}$$

$$X = 2.5$$

$$X = 1.5$$

$$3(x-3)^2 = 21 + 27$$

$$3(x-3)^2 = 48$$

$$(x-3)^2 = 16$$

$$x-3 = \pm 4$$

$$16) 2k^2 - 16k + 30 = 0$$

$$x = 3 \pm 4$$

$$2(k^2 - 8k) = -30$$

$$2(k-4)^2 = -30 + 32$$

$$2(k-4)^2 = 2$$

$$\begin{array}{|c|} \hline K = 5 \\ \hline K = 3 \\ \hline \end{array}$$

$$(k-4)^2 = 1 \quad K-4 = \pm 1 \quad K = 4 \pm 1$$

$$17) 7x^2 + 14x - 56 = 0$$

$$7(x^2 + 2x) = 56$$

$$(x+1) = \pm 3$$

$$7(x+1)^2 = 56 + 7$$

$$x = -1 \pm 3$$

$$7(x+1)^2 = 63$$

$$X = -4$$

$$(x+1)^2 = 9$$

$$X = 2$$

$$19) 8k^2 - 16k - 64 = 0$$

$$8(k^2 - 2k) = 64$$

$$8(k-1)^2 = 64 + 8$$

$$K = 1 \pm 3$$

$$8(k-1)^2 = 72$$

$$K = 4$$

$$(k-1)^2 = 9$$

$$K = -2$$

$$k-1 = \pm 3$$

$$21) 2a^2 - 4a - 6 = 0$$

$$2(a^2 - 2a) = 6$$

$$a = 1 \pm 2$$

$$2(a-1)^2 = 6 + 2$$

$$a = 3$$

$$2(a-1)^2 = 8$$

$$a = -1$$

$$(a-1)^2 = 4$$

$$a-1 = \pm 2$$

$$23) 7x^2 + 14x - 21 = 0$$

$$7(x^2 + 2x) = 21$$

$$x+1 = \pm 2$$

$$7(x+1)^2 = 21 + 7$$

$$x = -1 \pm 2$$

$$7(x+1)^2 = 28$$

$$\begin{array}{|c|} \hline X = 1 \\ \hline X = -3 \\ \hline \end{array}$$

$$(x+1)^2 = 4$$

$$25) 4p^2 - 8p - 60 = 0$$

$$4(p^2 - 2p) = 60$$

$$p = 1 \pm 4$$

$$4(p-1)^2 = 60 + 4$$

$$p = 5$$

$$4(p-1)^2 = 64$$

$$p = -3$$

$$(p-1)^2 = 16$$

$$p-1 = \pm 4$$

$$18) 3b^2 - 12b - 15 = 0$$

$$3(b^2 - 4b) = 15$$

$$3(b-2)^2 = 15 + 12$$

$$b = 2 \pm 3$$

$$3(b-2)^2 = 27$$

$$(b-2)^2 = 9$$

$$b-2 = \pm 3$$

$$b = 5$$

$$b = -1$$

$$20) 10x^2 + 20x - 38 = 0$$

$$10(x^2 + 2x) = 38$$

$$10(x+1)^2 = 38 + 10$$

$$10(x+1)^2 = 48$$

$$(x+1)^2 = 4.8$$

$$x+1 = \pm \sqrt{4.8}$$

$$X = -1 \pm \sqrt{4.8}$$

$$22) 2x^2 + 20x + 22 = 0$$

$$2(x^2 + 10x) = -22$$

$$2(x+5)^2 = -22 + 50$$

$$2(x+5)^2 = 28$$

$$(x+5)^2 = 14$$

$$x+5 = \pm \sqrt{14}$$

$$X = -5 \pm \sqrt{14}$$

$$24) x^2 - 20x + 36 = 0$$

$$(x-10)^2 = -36 + 100$$

$$(x-10)^2 = 64$$

$$x-10 = \pm 8$$

$$X = 18$$

$$x = 10 \pm 8$$

$$X = 2$$

$$26) b^2 + 4b - 18 = 0$$

$$(b+2)^2 = 18 + 4$$

$$(b+2)^2 = 22$$

$$b+2 = \pm \sqrt{22}$$

$$b = -2 \pm \sqrt{22}$$

$$27) 10n^2 + 20n - 30 = 0$$

$$10(n^2 + 2n) = 30 \quad n+1 = \pm 2$$

$$10(n+1)^2 = 30 + 10$$

$$10(n+1)^2 = 40$$

$$(n+1)^2 = 4$$

$$29) n^2 - 18n - 22 = 0$$

$$(n-9)^2 = 22 + 81$$

$$(n-9)^2 = 103$$

$$n-9 = \pm \sqrt{103}$$

$$n = 9 \pm \sqrt{103}$$

$$\begin{array}{l} n=1 \\ n=-3 \end{array}$$

$$31) x^2 - 8x - 6 = 3$$

$$(x-4)^2 = 9 + 16$$

$$(x-4)^2 = 25$$

$$x-4 = \pm 5$$

$$x = 4 \pm 5$$

$$\begin{array}{l} x=9 \\ x=-1 \end{array}$$

$$33) n^2 - 16n + 17 = -8$$

$$(n-8)^2 = -25 + 64$$

$$(n-8)^2 = 39$$

$$n-8 = \pm \sqrt{39}$$

$$n = 8 \pm \sqrt{39}$$

$$35) 7x^2 + 14x - 54 = 2$$

$$7(x^2 + 2x) = 56 \quad x = -1 \pm 3$$

$$7(x+1)^2 = 63$$

$$(x+1)^2 = 9$$

$$x+1 = \pm 3$$

$$\begin{array}{l} x=2 \\ x=-4 \end{array}$$

$$37) 9n^2 - 18n - 50 = 3$$

$$9(n^2 - 2n) = 53$$

$$9(n-1)^2 = 53 + 9$$

$$9(n-1)^2 = 62$$

$$(n-1)^2 = 6.9$$

$$n = 1 \pm \sqrt{6.9}$$

$$39) p^2 - 4p - 63 = -3$$

$$(p-2)^2 = 60 + 4$$

$$(p-2)^2 = 64$$

$$p-2 = \pm 8$$

$$p = 2 \pm 8$$

$$\begin{array}{l} p=10 \\ p=-6 \end{array}$$

$$28) 5n^2 - 10n - 100 = 0$$

$$5(n^2 - 2n) = 100$$

$$5(n-1)^2 = 100 + 5$$

$$5(n-1)^2 = 105$$

$$(n-1)^2 = 21$$

$$n-1 = \pm \sqrt{21}$$

$$n = 1 \pm \sqrt{21}$$

$$30) 2m^2 + 8m - 3 = 0$$

$$2(m^2 + 4m) = 3$$

$$2(m+2)^2 = 3 + 8$$

$$2(m+2)^2 = 11$$

$$(m+2)^2 = 5.5$$

$$m = -2 \pm \sqrt{5.5}$$

$$32) 10k^2 - 20k - 82 = -2$$

$$10(k^2 - 2k) = 80$$

$$10(k-1)^2 = 80 + 10$$

$$10(k-1)^2 = 90$$

$$(k-1)^2 = 9$$

$$k-1 = \pm 3$$

$$k = 1 \pm 3$$

$$k = 4$$

$$k = -2$$

$$34) 10x^2 - 20x - 22 = 2$$

$$10(x^2 - 2x) = 24$$

$$10(x-1)^2 = 24 + 10$$

$$10(x-1)^2 = 34$$

$$(x-1)^2 = 3.4$$

$$x-1 = \pm \sqrt{3.4}$$

$$x = 1 \pm \sqrt{3.4}$$

$$36) p^2 - 14p + 4 = 5$$

$$(p-7)^2 = 1 + 49$$

$$(p-7)^2 = 50$$

$$p-7 = \pm \sqrt{50}$$

$$p = 7 \pm 5\sqrt{2}$$

$$38) n^2 + 10n - 58 = 4$$

$$(n+5)^2 = 62$$

$$n = -5 \pm \sqrt{62}$$

$$40) m^2 + 10m - 17 = -6$$

$$(m+5)^2 = 11 + 25$$

$$(m+5)^2 = 36$$

$$m+5 = \pm 6$$

$$m = -5 \pm 6$$

$$\begin{array}{l} m=1 \\ m=-11 \end{array}$$