

## Day 3 – Comparing Multiple Representations

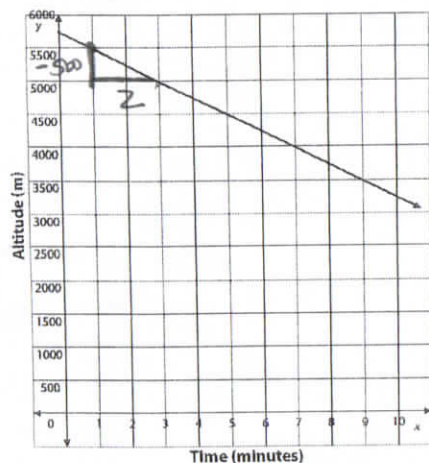
(Tasks for Day 3 have been adopted from Walch Coordinate Algebra &amp; Analytic Geometry Resources)

When comparing different functions, it is helpful to think about what characteristics apply most to each type of function. Use the chart below to help you compare the most important features of each function.

Linear	Quadratic	Exponential
Slope (Rate of Change)	Vertex	Growth/Decay Rate
Y-intercept (Starting Point)	X-intercepts	Y-intercept (Starting Point)
	Y-intercepts	Asymptote

## Scenario 1:

Two airplanes are in flight. The function  $f(x) = 400x + 1200$  represents the altitude,  $f(x)$  of one airplane after  $x$  minutes. The graph below represents the altitude of the second airplane.



- a. Compare the starting altitudes of both airplanes.

$y_{int}$   
Graph  $\rightarrow g(x)$

- b. Compare the rates of changes of both airplanes.

$f(x)$  Slope

①  $f(x) = 400x + 1200$

②  $g(x) = -375x + 5750$

## Scenario 2: Compare the two functions below:

## Function A

A local newspaper began with a circulation of 1,300 readers in its first year. Since then, its circulation has increased by 150 readers per year.

## Function B

The function  $g(x) = 225x + 950$  represents the circulation of another newspaper where  $g(x)$  represents total subscriptions and  $x$  represents the number of years since its first year.

$$y = 150x + 1300$$

G slope; B  
greatest reader start: A

## Scenario 3: Compare the two functions below:

## Function A

The value of a car in dollars,  $f(x)$  depreciates after each year,  $x$ . The following table shows the value of a car for each of the first 4 years after it was purchased.

$x$	$f(x)$
0	22,450
1	19,307
2	16,604.02
3	14,279.46
4	12,280.33

## Function B

The value of a second car is modeled by the equation  $g(x) = 19,374(1-0.16)^x$ , where  $g(x)$  represents the value of the car  $x$  years after the date it was purchased.

$$g(x) = 19,374(0.84)^x$$

$$r = \frac{\text{2nd}}{\text{1st}} = 0.86$$

Exponential =  $y = a(b)^x$

$$y = 22,450(0.86)^x$$

higher  
start value

A  
decaying faster

**Scenario 4:**

Through comparison shopping, you have obtained the yearly insurance rate quotes from three car insurance companies. The insurance rate quote is given as a function of the age of the customer. Using the information presented below, which company has the lowest rate for a person in their fifties?

- Wreck-for-Less Insurance:  $R(x) = 0.87x^2 - 91.48x + 3185$ , where  $R(x)$  is the yearly premium in dollars and  $x$  is the age in years?  $R(50) = 786$
- Careless Insurance: A 51 year old customer pays a yearly premium of \$900. For every 5 year increase or decrease in age, the customer is charged an additional \$300, then \$500, then \$700, and so on.
- Fender & Bender Insurance:

Age of customer	Under 24	25-29	30-39	40-49	50-59	Over 59
Insurance rate (\$)	1,750	1,500	1,125	916	875	916

x	y
50-55	900
56-60	1200
61-65	1700
66-70	2400

Additionally, if you are given a table or pattern, you can use those to create an equation using a regression model on your graphing calculator. You must know what type of function each scenario represents to use a regression model.

**Scenario 1:**

You are considering investing \$5,000 in one of two mutual funds. The first fund will pay \$500 each year. The second fund is predicted to have end of year balances as shown in the following table.

x (year)	0	1	2	3	4	5
f(x) (\$)	5,000	5,200	5,500	5,900	6,400	7,000

- a. Identify what type of function each fund is and why:

Type: Linear Why: \$500 each year

Type: Quadratic Why: Common 2nd Difference

4: LinReg  
5: QuadReg  
0: ExpReg

- b. Create an equation to model both mutual funds:

Mutual Fund A:  $y = 500x + 5000$

Mutual Fund B:  $y = 50x^2 + 150x + 5000$

- c. Which fund should you choose if you want to withdraw your money after 5 years and why?

L \*  
7500

Q  
7000

- d. Which fund should you choose if you want to withdraw your money after 10 years and why?

L  
10,000

\* Q  
11,500

**Scenario 2:**

Suppose you are offered a position at a prestigious company. You may choose how your salary is paid.

- Option 1 is described by the quadratic equation  $S(x) = 2500x^2 + 2500x + 60,000$  where  $x$  is the number of years you are with the company and  $S(x)$  is the yearly salary in dollars.
- Option 2 has a starting salary of \$35,000, but you get a 25% raise each year. (Equation: Exponential)  $35000(1 + .25)^x$

- a. Identify what type of function each fund is and create an equation to model both mutual funds:

Type: Quad. Why: Given

Type: Exponential Why: Raise in %

b. If you plan to work for one of these companies for 5 years, which option should you choose?

\* Q  
135,000

E  
106,811.52

c. If you plan to work for one of these companies for 30 years, which option should you choose?

Q  
\$2,385,000

\* E  
\$28,272,774.84

d. When does Option 2's salary exceed Option 1's salary?

\* Create a table  
Year 11

### Scenario 3:

An emergency meeting has been called on Capitol Hill in Washington DC. As a newly elected representative for the State of Georgia, you need to find a way to travel to Washington. No flights are available at a time that fits your schedule, but there are three trains that you could take. The distance between Atlanta and Washington DC is 639 miles.

- 7:30 PM 8 • Train 1: Train is a nonstop train that leaves at 10 AM and travels 66 miles per hour.
- 7:30 PM 8 • Train 2: Train also leaves at 10 AM, but has a few stops that slow it down. The total distance traveled each hour is shown in the table below.

t	0	1	2	3	4	5	6	7	8	9	10
d(t)	0	48	100	156	216	280	348	420	496	576	660

- Train 3: Train leaves at noon. The distance traveled for Train 3 can be modeled by  $d(t) = 6t^2 + 32t$ , where  $t$  is the time in hours beginning at noon and  $d(t)$  is the distance traveled in miles.

Hour 8pm = 640 miles ✓ arrives

a. Is it possible to determine which train will arrive in Washington first?

1 2 3

b. Which train should you select if you want to minimize time spent on the train?

1

### Scenario 4:

Today is the day of the Speedy Snail Race. Garrett's snail is he acknowledged favorite, but there is expected to be competition from Patrick's and Sandra's snails. The function below describes each snail's distance from the starting line, in inches, with  $t$  representing the time in minutes since the race started. If the race is 300 inches long, whose snail would be predicted to win based off their representations below:

- Garrett's snail:  $G(t) = 1.5t$
- Sandra's snail:  $S(t) = 2t^2 - t$
- Patrick's snail:

Q

t	1	2	3	4	5	6
P(t)	2	6	12	20	30	42

$x^2 + x$

4 6 8 10

a. Who places 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> and how long does it take each snail to cross the finish line?

G: 13 minutes 1<sup>st</sup>

S: 14-15 minutes 2<sup>nd</sup>

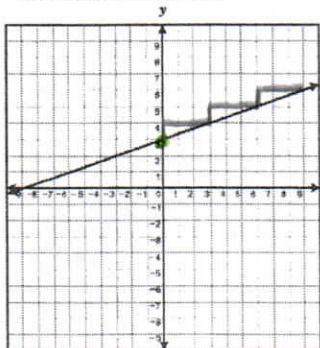
P: 17 minutes 3<sup>rd</sup>



# Comparing Linear and Exponential Functions

## Homework

1. The functions  $f(x)$  and  $g(x)$  are described below. Compare the rate of change and intercepts of each. What do you notice about the two functions?



$(0, 3)$   
greater

$$\frac{1}{3}$$

\* slopes are equal

\* graph is moved up 6 from the table

x	g(x)
-3	-4
0	-3
3	-2
6	-1

$$\frac{-3 - (-4)}{0 - (-3)} = \frac{1}{3}$$

2. The gym offers 3 membership plans.

**Pay As You Go:** \$6 each time you work out

**Regular Deal:** \$50 per month plus \$2 each time you work out

**Unlimited Deal:** \$100 per month for unlimited use.

6x

$$2x + 50$$

100

What does the y-intercept of each function represent? monthly cost

Pay as you go is the cheapest plan until what number of visits is reached?

	P	R
Month 13	\$78	\$76
Month 17	\$102	100

3. Supply is modeled by the linear function  $f(x) = 0.3x + 100$ , where  $f(x)$  represents the price per tablet in dollars and  $x$  represents the number of tablets. Demand is modeled in the table below, where  $g(x)$  represents the price per tablet in dollars and  $x$  represents the number of tablets.

x	g(x)
100	490
300	370
500	250
600	190

$$\frac{370 - 490}{300 - 100} = \frac{-120}{200}$$

Find the rate of change of each function. Which of the two lines is the steepest?

$f(x)$  rate: 0.3      \*  $g(x)$  rate: -0.6

Find the equation for  $g(x)$ , and find the intersection of the two lines. What does this point represent?

$$g(x) = -0.6x + 550 \quad f(x) = 0.3x + 100$$

$$(1250, 475)$$

What happens if the supply exceeds the demand?

# Algebra I Unit 5: Comparing & Constrasting Functions

4. Compare the y-intercept and rate of change for each function. Based on this information, which function would you choose?

**Function A:** A rental store charges \$40 to rent a steam cleaner and \$4 for each additional hour.

**Function B:**

Hours (x)	Total cost (g(x))
3	46
4	53
5	60
6	67

$$y = 4x + 40$$

Choose

$$y = 7x + 45$$

higher rent cost

higher charge per hour

5. Ryan has invested \$50,000 in the bank at 2% simple interest. The function to represent his money in the bank is  $R(x) = 50,000(1.015)^x$ .

Jamie has invested \$20,000 in the bank at 3% simple interest. The function to represent his money in the bank is  $J(x) = 20,000(1.03)^x$ .

Compare the rate of change and y-intercepts for the first 20 years in the bank.

	R(x)	ROC	J(x)	ROC
greater start in investment	(0, 50000)	807.15	(0, 20000)	806.1
	(20, 61343)	*increasing investment faster	(20, 30123)	

Ryan and Jamie are both 50 years old. Do you think Jamie's savings will ever catch Ryan if neither one of them touches the money that is in their accounts?

50 yrs. Ryan still has more  
Not until year 63 Jamie

6. Compare the rate of change and y-intercepts of each exponential function over the interval [0, 4].

**Function A:** The value of a car in dollars,  $f(x)$ , depreciates after each year,  $x$ . The following table shows the value of a car for each of the first 4 years after it was purchased.

$$\text{ROC: } \frac{12,280.33 - 22,450}{4} = -2542.42$$

x	f(x)
0	22,450
1	19,307
2	16,604.02
3	14,279.46
4	12,280.33

$$r = \frac{19,307}{22,450} = .86$$

Starts w/ more \$

**Function B:** The value of a second car is modeled by the equation  $g(x) = 19,375(0.84)^x$ , where  $g(x)$  represents the value of the car  $x$  years after the date it was purchased.

$$\text{ROC: } \frac{9646.26 - 19375}{4} = -2432.19$$

$$r = .84 \rightarrow \text{smaller ratio}$$

Find the common ratio for both functions. Which one is smaller? How does it compare to the rate of change for that function?

Function B also has a smaller ROC